MIP 2007:
Workshop on Mixed Integer Programming
July 30 – August 2, 2007

Enhanced cutting planes for a class of binary multi-level optimization algorithms

J. Cole Smith
Dept. of Industrial and Systems Engineering
The University of Florida
P.O. Box 116595
Gainesville, FL 32611-6595
USA
cole@ise.ufl.edu

Abstract

Bi-level optimization algorithms have been developed for use in a variety of problems, including network design, scenario-based stochastic programming, and network interdiction applications. Benders-decomposition-based algorithms divide the decision-making process into a first-stage master problem wherein "linking" variable values are determined, after which a set of separable second-stage subproblems are solved. The master problem determines an optimistic assessment of the second-stage objective, after which Benders cutting planes refine this assessment by providing "cause-and-effect" relationships between the linking decisions and second-stage objective function. However, these cutting planes are often too weak to be used for practical problems in which binary variables appear in the first stage.

The tightening of these cutting planes by a master problem reformulation process is the subject of this talk. The author has recently encountered a surprising feature of some bi-level problems, in which an exponential number of nondominated cutting planes (using Benders decomposition) can be generated at each stage. However, by (a) reformulating the master problem in a (polynomially sized) higher-dimensional space, and (b) linking the master problem and subproblems via these new variables, it is possible to imply all of the exponentially many cutting planes in the original variable space with a single inequality in the new variable space. This result implies that previous problems that were thought to be impossible to solve via exact optimization may in fact be solvable within reasonable computational limits.

This talk will provide a few examples of this reformulation strategy and will discuss the computational implications of the strategy. It will also describe current progress is seeking the relationship of this procedure to existing reformulation techniques, toward a unified(rather than ad-hoc) method of applying the proposed master problem reformulation technique.