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Polyhedral aspects of symmetry breaking in integer programming

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Abstract

Some very natural integer programming models are highly symmetric in the sense that there is a group acting on the feasible solutions such that the objective function remains constant along each orbit. Classical examples are models with an assignment substructure, e.g., for vertex-coloring or graph partitioning.

On such models, standard branch-and-cut algorithms basically do the same work in many subtrees that are equivalent via the symmetry. Moreover, due to the barycenters of the orbits being feasible fractional solutions, the linear programming bounds tend to be weak. Therefore, one tries to remove the symmetry from the models by adding additional inequalities that preferably leave feasible only one single representative of each orbit.

In this talk, we report on our investigations of some basic polyhedra arising in this context. Given a set of 0/1-matrices (like all 0/1-matrices of a fixed size that have exactly one one-entry per row) these orbitopes are the convex hulls of those matrices that are lexicographically maximal in their orbit. Moreover, we describe a procedure(called orbitopal fixing) to utilize the knowledge on orbitopes very efficiently at every branch-and-cut node without explicitly adding inequalities to the model. Computational experiments with the graph partitioning problem show that the methods significantly improve the performance of state-of-the-art branch-and-cut codes.

Join work with Marc E. Pfetsch (Zuse-Institute Berlin) and Matthias Peinhardt(Otto-von-Guericke University Magdeburg).