Certificate of infeasibility and cutting planes from lattice-point-free polyhedra

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Joint work with K. Andersen (Copenhagen), R. Weismantel (Magdeburg)

Outline

- Split Cuts
- Lattice-Point-Free Polyhedra
- Integral Farkas Lemma for Systems with Inequalities
- Cutting Planes from Lattice-Point-Free Polyhedra
- Conclusion

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Split cuts

The algebra

Based on a disjunction

$$\pi^T x \leq \pi_0 \quad \text{or} \quad \pi^T x \geq \pi_0 + 1$$

is valid for $x \in \mathbb{Z}^n$ when π, π_0 are integer.

The geometry

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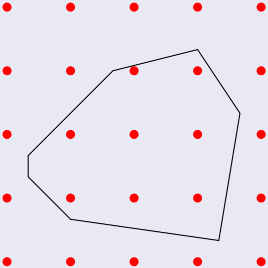

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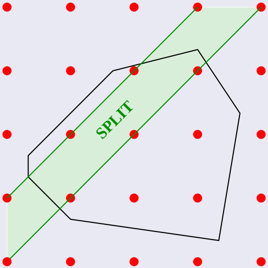

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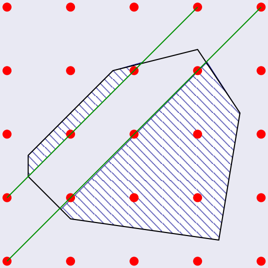

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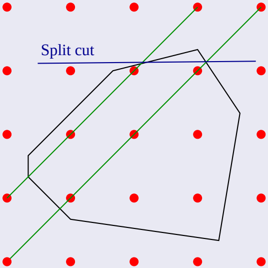

The split closure

Consider a polyhedron $P \subseteq \mathbb{R}^n$, the intersection of all split cuts of P is called the (first) split closure of P, denoted by $\mathrm{SC}(P)$.

Some previous results

- Cook, Kannan, Schrijver [1990] The split closure is a polyhedron
- Lift-and-project, Chvátal-Gomory cuts are split cuts
- Nemhauser, Wolsey [1988] MIR inequalities are split cuts and MIR closure and split closure are equivalent
- Cook, Kannan, Schrijver [1990] The number of rounds of split cuts to apply to obtain the integer hull of a polyhedron might be infinite
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  Split cuts are intersection cuts [Balas 1971]

More about split cuts

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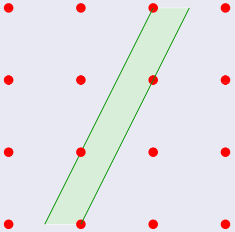

A basic split set in $\mathbb{R}^2$ is a lattice-point-free polyhedron

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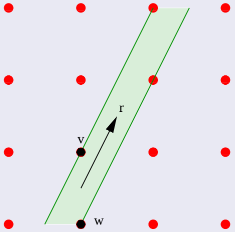

$\text{conv}\{v, w\} + \text{span}\{r\}$

Towards high-dimensional splits

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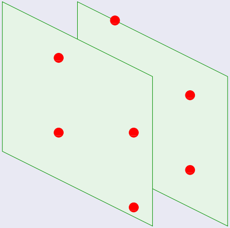

A basic split set in $\mathbb{R}^3$ is a lattice-point-free polyhedron

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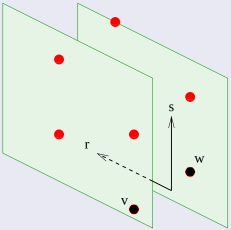

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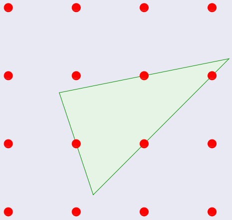

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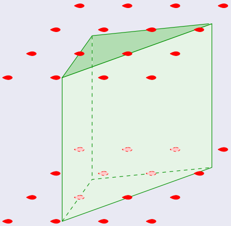

A triangle in $\mathbb{R}^2$ can be lattice-point-free
It can be lifted to a lattice-point-free polyhedron in $\mathbb{R}^3$

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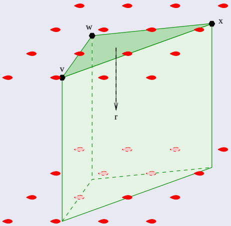

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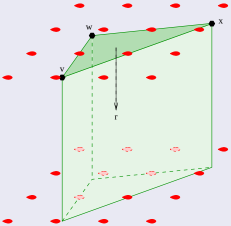

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Definition of the split dimension

A lattice-point-free polyhedron $P \subseteq \mathbb{R}^n$ can be written as

$$P = \text{conv}\{v^1, \ldots, v^p\} + \text{cone}\{w^1, \ldots, w^q\} + \text{span}\{r^1, \ldots, r^{n-d}\}.$$

The split-dimension of P is d.

The continuous Farkas Lemma [Farkas, 1902]

Let $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$,

$$\begin{array}{l} Ax \leq b \\ x \geq 0 \\ x \in \mathbb{R}^n \end{array} \quad \text{is empty if and only if} \quad \begin{array}{l} y^T A \geq 0 \\ y^T b < 0 \\ \text{for some } y \in \mathbb{R}^m. \end{array}$$

Example

$$\begin{array}{lrl} (1) & 10x_1 + 14x_2 & \leq 35 \\ (2) & -x_1 + x_2 & \leq 0 \\ (3) & -x_2 & \leq -2 \end{array}$$

A certificate of infeasibility

$$y = (1 \quad 8 \quad 21)^T$$

$$\begin{array}{lrl} (1) & 10x_1 + 14x_2 & \leq 35 \\ 8(2) & -8x_1 + 8x_2 & \leq 0 \\ 21(3) & -21x_2 & \leq -42 \end{array}$$

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Classical Integral Farkas Lemma

The Integral Farkas Lemma [Kronecker 1884]

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Example

$$\begin{array}{ll} (1) & 3x_1 + x_2 - 5x_3 + x_4 - 7x_5 = 1 \\ (2) & 7x_1 - 3x_2 - 3x_3 - 2x_4 + 5x_5 = 5 \\ (3) & 2x_1 + x_2 + x_3 + 6x_4 = 1 \end{array}$$

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$$y = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \end{pmatrix}$$

$$\begin{array}{lrcl} \frac{1}{3}(1) & x_1 + \frac{1}{3}x_2 - \frac{5}{3}x_3 + \frac{1}{3}x_4 - \frac{7}{3}x_5 &=& \frac{1}{3} \\ \frac{2}{3}(2) & \frac{14}{3}x_1 - 2x_2 - 2x_3 - \frac{4}{3}x_4 + \frac{10}{3}x_5 &=& \frac{10}{3} \\ \frac{2}{3}(3) & \frac{4}{3}x_1 + \frac{2}{3}x_2 + \frac{2}{3}x_3 + 4x_4 &=& \frac{2}{3} \end{array}$$

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\end{array}$$

$Ax = b$	$\{v^*\} + \text{span}\{w^1, \dots, w^d\}$
$y^T A$	subset of $\text{span}\{w^1, \dots, w^d\}^\perp$
$y^T b \notin \mathbb{Z}$	there exists $\pi \in \text{span}\{w^1, \dots, w^d\}^\perp \cap \mathbb{Z}^n$ with $\pi^T v^* \notin \mathbb{Z}$.

Equivalent to say that $L = \{\lfloor \pi^T v^* \rfloor \leq \pi^T x \leq \lceil \pi^T v^* \rceil\}$ contains $Ax = b$ in its interior.

Existence of a split proving that $Ax = b \cap \mathbb{Z}^n = \emptyset$

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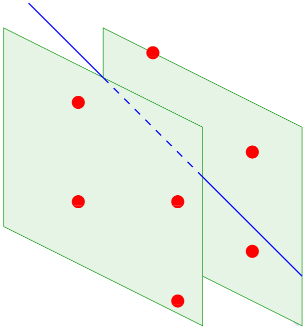

Natural extension for one range inequality

Integral Farkas Lemma with one range inequality [Andersen, L. , Weismantel 2007]

$$\begin{array}{l} Ax = b \\ l \le cx \le u \\ x \in \mathbb{Z}^n \end{array} = \emptyset \quad \text{iff} \quad \exists y \in \mathbb{Q}^m, z \in \mathbb{Q}_+ \text{ with } \begin{array}{l} (y^T\ z)\begin{pmatrix} A \\ c \end{pmatrix} \in \mathbb{Z}^n \\ [y^T b + zl, y^T b + zu] \cap \mathbb{Z} = \emptyset \end{array}$$

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$$\begin{array}{rrr} (1) & & 2x_1 + x_2 + 3x_3 - x_4 = 3 \\ (2) & & 6x_1 - x_2 - 2x_3 + x_4 = 5 \\ (3) & 5 \leq & 4x_2 + x_3 - 4x_4 \leq 8 \end{array}$$

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The certificate

$$y = \begin{pmatrix} \frac{2}{5} & \frac{1}{5} \end{pmatrix}, z = \frac{1}{5}$$

$$\begin{array}{lll} \frac{2}{5}(1) & \frac{6}{5} = & \frac{4}{5}x_1 + \frac{2}{5}x_2 + \frac{6}{5}x_3 - \frac{2}{5}x_4 = \frac{6}{5} \\ \frac{1}{5}(2) & 1 = & \frac{6}{5}x_1 - \frac{1}{5}x_2 - \frac{2}{5}x_3 + \frac{1}{5}x_4 = 1 \\ \frac{1}{5}(3) & 1 \le & \frac{4}{5}x_2 + \frac{1}{5}x_3 - \frac{4}{5}x_4 \le \frac{8}{5} \end{array}$$

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$$\sum \quad \frac{16}{5} \le 2x_1 + x_2 + x_3 - x_4 \le \frac{19}{5}$$

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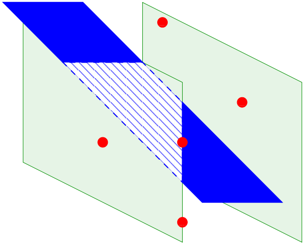

Idea

$$\begin{aligned} Ax &= b \\ Cx &\le d \\ x &\in \mathbb{Z}^n \end{aligned} \qquad (1)$$

The bigger rank(C), the more complicate the certificate of infeasibility.
(1) is infeasible if and only if $\{Ax = b, Cx \le d\}$ is contained in the interior of a lattice-point-free polyhedron of split-dimension equal to rank(C).

Integral Farkas Lemma for Systems with Equalities and Inequalities

[Andersen, L. , Weismantel 2007]
A certificate of infeasibility of (1) is an integral infeasible linear system (derived from the rows of (1)) with as many variables as rank(C).

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Example with rank$(C) = 2$

$$
\begin{array}{llr}
(1) & x_1 + 2x_2 + 3x_3 & = 0 \\
(2) & -3x_1 + 4x_2 & \leq 0 \\
(3) & -x_1 - 2x_2 & \leq -3 \\
(4) & 2x_1 - x_2 & \leq 5
\end{array}
$$

A certificate

$$
\begin{array}{ll}
\frac{1}{3}(1) + \frac{1}{12}(2): & x_2 + x_3 \leq -\frac{1}{12}x_1 \\
\frac{1}{3}(1) - \frac{1}{6}(3): & x_2 + x_3 \geq -\frac{1}{2}x_1 + \frac{1}{2} \\
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It is a system with 2 variables and 3 inequalities

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y \leq -\frac{1}{12}x_1 \qquad y \geq -\frac{1}{2}x_1 + \frac{1}{2} \qquad y \geq \frac{1}{3}x_1 - \frac{5}{3}
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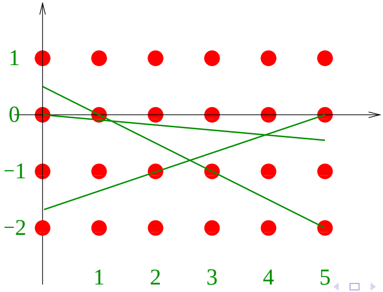

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We can rewrite the system using 2 variables corresponding to v^1 and v^2 respectively.

Final System

Theorem [Andersen, L. , Weismantel 2007]

Let $A \in \mathbb{Z}^{m\times n}$, $C \in \mathbb{Z}^{p\times n}$ with $\text{rank}(C) = L$.

$$\begin{aligned} Ax &= b \\ Cx &\le d \\ x &\in \mathbb{Z}^n \end{aligned}$$

is empty if and only if

- $\exists y^1, \ldots, y^t \in \mathbb{Q}^m \times \mathbb{Q}^p_+$
- $\exists L$ linearly independent $v^i \in \mathbb{Z}^n$ such that

$$(y^k)^T \begin{bmatrix} A \\ C \end{bmatrix} = \sum_{i=1}^{l} \lambda_i^k v^i \quad \in \mathbb{Z}^n \text{ with } \lambda_i^k \in \mathbb{Z}$$

- the system in variables z (representing $(v^i)^T x$)

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Some remarks about the lemma

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- For $\text{rank}(C) = 2$, the certificate is made of 3 or 4 inequalities
  Follows from [Andersen, L., Weismantel, Wolsey, IPCO2007]
- For $\text{rank}(C) \geq 3$, the number of inequalities in the certificate can be arbitrarily large
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  The feasibility problem $\{Ax = b, Cx \leq d\}$ where rank(C) is fixed is in co-NP.

  The proof follows from the fact that IP in fixed dimension is in P ([Lenstra 1983]) and that any infeasible IP in n variables is also infeasible on 2^n constraints ([Doignon 1973]).
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The algebra

Let $P \subseteq \mathbb{R}^{n+m}$ be a polyhedron and $L \subseteq \mathbb{R}^n$ be a lattice-point-free polyhedron. We define a set of cuts, valid for $\{(x, y) \in \mathbb{R}^{n+m} | x \in P \cap \mathbb{Z}^n\}$ as

$$\text{cuts}_P(L) = \text{conv}\{(x, y) \in \mathbb{R}^{n+m} | (x, y) \in P \text{ and } x \notin \text{int}(L)\}.$$

The geometry

Using the lattice-point-free polyhedra to generate cuts

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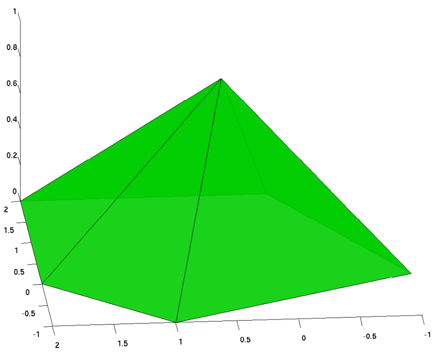

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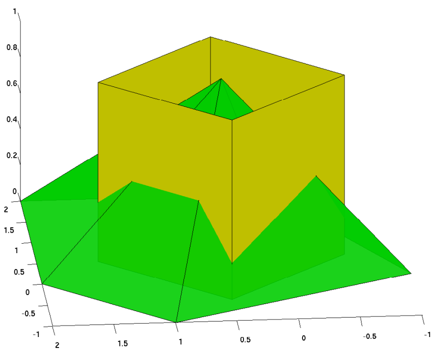

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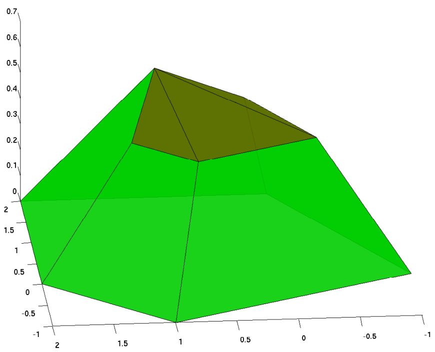

The high-dimensional split closure

Definition

The *d-dimensional split closure* of P is the set of points in the intersection of all high-dimensional split cuts obtained from P with a split-dimension less or equal to d.

Open question

Is the d-dimensional split closure of a polyhedron a new polyhedron ?

Cook, Kannan, Schrijver example [1990]

Can be solved in one iteration by a 2-dimensional split cut

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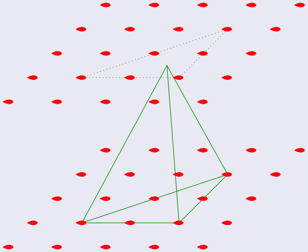

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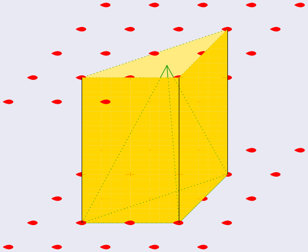

Family of polyhedra of dimension $n+1$ with an infinite n-dimensional split rank

Constructed in the same way :

- a n-dimensional lattice-point-free polyhedron with integer points on the interior of each facet
- lifted by an ϵ in a $(n+1)$th variable

$$P = \text{conv}\{(ne_1, 0), (ne_2, 0), \ldots, (ne_n, 0), (\frac{1}{2}\underline{1}, \epsilon)\}$$

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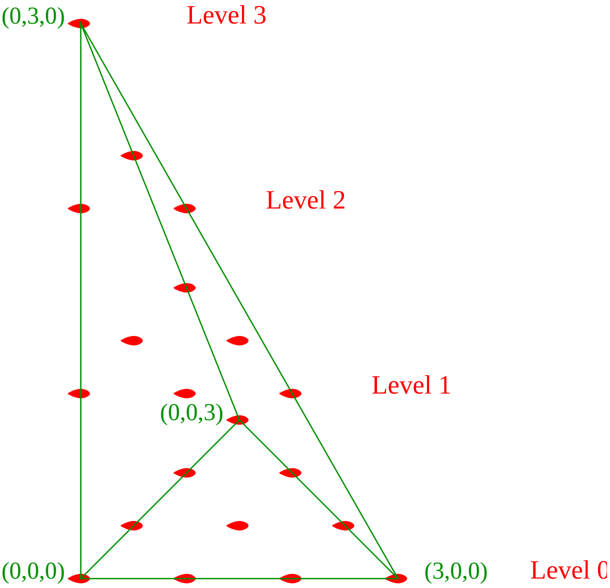
(0,3,0)
Level 3
Level 2
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(0,0,0)
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Level 0

Conclusions

- Lattice-point-free polyhedra provide a new geometric interpretation of cutting planes
- How to use them in practice ? Closed form formulae ?
- What is the split rank of the cuts generated (sometimes infinite but not always) ?
- Allows us to obtain a cutting plane algorithm that runs in finite time ?
- What about the fact that the high-dimensional split closure is a polyhedron ?

Conclusions

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