Konrad-Zuse-Zentrum für Informationstechnik Berlin
Abteilung Optimierung

Solving non-linear integer programs arising in mine production planning

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Production scheduling for an open pit mine with a stockpile

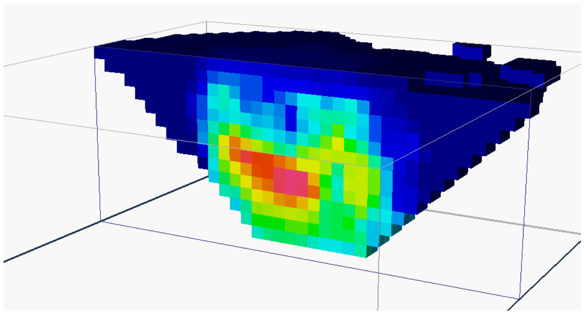
Goal Maximize net present value

Decisions

- Which block is mined when?
- What material is processed immediately?
- What material is stockpiled?
- What fraction of stockpile is processed when?

Constraints

- Block mining order
- Mining and processing capacities
- ***Mixing of material in stockpile***

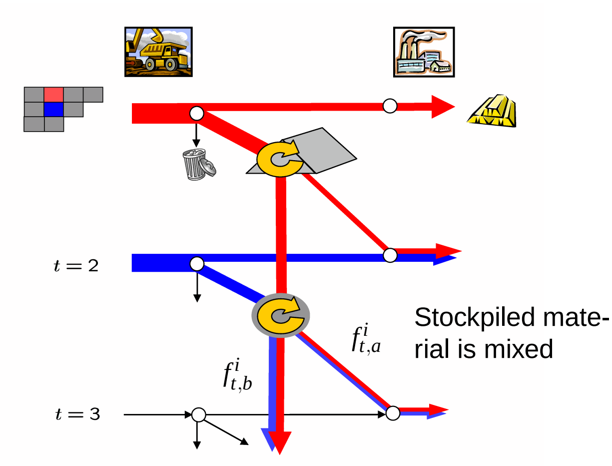


Block model of an open pit mine
blue = low grade ore, red = high grade ore

Time-indexed MINLP Model

Standard mixed-integer linear model

- Binary variables for block mining starts
- Continuous var's for block-wise material flows
- Linear constraints for capacities, mining order, and material flow conservation



+ Non-linear non-convex mixing constraints

$$f^i_{t,a}\, f^j_{t,b} = f^j_{t,a}\, f^i_{t,b} \quad \forall t, i, j$$

Solution approach

Branch-and-bound with linear outer approximations

(1) **Relax integrality and non-linear constraints.**

- LP $\Rightarrow$ efficiently computable upper bound.

(2) **Iteratively branch on integer-infeasible variables or violated non-linear constraints.**

- New subproblems are defined by variable fixing or by additional linear constraints. $\Rightarrow$ All subproblems are LPs.
- Yields integer-feasible solution candidates.
- Narrows violation of non-linear constraints (arbitrarily close to 0).

(3) **Postprocess integer-feasible candidates.**

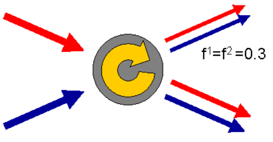
- Simple modification of material flows to turn integer-feasible candidates into fully feasible solutions that satisfy also the non-linear mixing constraints.

$\Rightarrow$ **Globally optimal solution**

$\Rightarrow$ **Fast** (better than a-priori discretization of non-linear constraints)

General mixing constraints

Homogeneously mixed multicommodity flow



Mixing constraints

$$f_a^i\, f_b^j = f_a^j\, f_b^i \quad \forall i,j = 1,\dots,k,\ a,b \in \delta^+(v)$$

or (with appropriate definition of division by 0)

$$\frac{f^1(A)}{f^1(B)} = \dots = \frac{f^k(A)}{f^k(B)} \quad \forall A,B \subset \delta^+(v) \qquad (*)$$

In specialized branch-and-bound algorithm (*) be treated as a single constraint.

Branching on violated mixing constraints

Violated mixing constraint

$$\exists t,i,j: \frac{f_{t,a}^i}{f_{t,a}^i + f_{t,b}^i} > \phi > \frac{f_{t,a}^j}{f_{t,a}^j + f_{t,b}^j}$$

out-of-stock vs. content-in-stock for commodity i

out-of-stock vs. content-in-stock for commodity j

Create new subproblems

Branch 1 out/content ratio $\leq \phi$	**Branch 2** out/content ratio $\geq \phi$
$(1-\phi)\, f_{t,a}^1 \leq \phi\, f_{t,b}^1$	$(1-\phi)\, f_{t,a}^1 \geq \phi\, f_{t,b}^1$
$(1-\phi)\, f_{t,a}^2 \leq \phi\, f_{t,b}^2$	$(1-\phi)\, f_{t,a}^2 \geq \phi\, f_{t,b}^2$
$\vdots$	$\vdots$
$(1-\phi)\, f_{t,a}^k \leq \phi\, f_{t,b}^k$	$(1-\phi)\, f_{t,a}^k \geq \phi\, f_{t,b}^k$

- Current solution is cut off and potential violation of mixing constraint is reduced.
- Subproblems are defined by adding linear constraints only.
- Branches simultaneously on all bilinear equations of mixing constraint.

Results

Mixing constraints can be handled efficiently by special branching + linear outer approximation

Problem	Approach	Upper bound	Solution	Gap (%)	Time (s)
Marvin	MILP relaxation (agg) + postprocessing	*9.29	5.43	41.5	~100
(85 blocks)	MILP with a-priori discretization	*7.00	6.94	1.0	»10000
	Specialized branching	7.04	6.94	1.4	10000
	Specialized branching + strengthening	7.00	6.94	1.0	2502
OB25	MILP relaxation (agg) + postprocessing	*5.51	4.40	18.7	~5500
(125 blocks)	MILP with a-priori discretization	*4.92	4.88	1.0	»10000
	Specialized branching	4.93	4.88	1.0	721
	Specialized branching + strengthening	4.93	4.88	1.0	1476

(* - optimal value of MILP relaxation, gaps w.r.t. optimal solution)

Stockpiles at Yandi mine, Australia.

Cooperation

The University of Melbourne

University of New South Wales

BHP Billiton