Xpress-MP performance

Solving Mixed Integer Programs with Xpress-MP

We constantly benchmark our optimization engines on public test sets and on customer instances.

**Speed Up on Internal Test Set**
The current release 2007A of the Xpress-Optimizer is
- 22% faster by geometric mean and
- 41% faster by arithmetic mean than 2006A on our internal test set.

**Solving Unsolved Instances from MIPLIB 2003**

The MIPLIB 2003 is a collection of 60 MIPLIBs of which many are hard-to-solve or remain unsolved until today.

With the latest improvements in Xpress-Optimizer, it was possible to
- solve 10 (out of 17) problems which where unsolved before Dec-2005.
- solve 5 new problems for the 1st time: atlanta-ip, ms98lp, protol, rd-rplusc-21 and split72.
- solve a1c1 at1 for the 1st time on a single computer
- find the 1st feasible solution for the stipld problem (500,766 solution which is within 3.5% of the optimum).
- find better solutions for all current 6 unsolved problems.

**Main MIP Components that Lead to the MIPLIB 2003 Improvements**

The following improvements were crucial to obtain these results.
- presolving
- node-to-node preprocessing
- cutting
- use of lift-and-project cuts
- better branching decisions
- improved strong branching estimates

Besides these, for some instances it was useful to search with a special strategy for a good solution and use this to restrict the search for the bound raising tree.

To be able to raise the bound faster, we explored the top of the tree with heavy strong branching for a certain number of nodes, applied a full presolve on the leaf nodes of this tree and continued the search from this point on.

The results in more detail are the following:

**Optimal Solutions**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Old Best Known</th>
<th>Xpress Improved</th>
<th>Gap</th>
<th>What's New</th>
</tr>
</thead>
<tbody>
<tr>
<td>atlanta-ip</td>
<td>10.90954377</td>
<td>9.00987812</td>
<td>9.3%</td>
<td>0.3%</td>
</tr>
<tr>
<td>mcp98lp</td>
<td>120.986991.06</td>
<td>111.6263192</td>
<td>9.4%</td>
<td>0.3%</td>
</tr>
<tr>
<td>protol</td>
<td>101</td>
<td>91.125999</td>
<td>9.5%</td>
<td>0.3%</td>
</tr>
<tr>
<td>rd-rplusc-21</td>
<td>777.62425</td>
<td>559.114037</td>
<td>5.5%</td>
<td>0.6%</td>
</tr>
<tr>
<td>split72</td>
<td>116.59</td>
<td>110.72363</td>
<td>5.5%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

**New Primal and Dual Bounds**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Old Best Known</th>
<th>Xpress Improved</th>
<th>Gap</th>
<th>What's New</th>
</tr>
</thead>
<tbody>
<tr>
<td>becker</td>
<td>116.59</td>
<td>114.36347</td>
<td>16%</td>
<td>0.9%</td>
</tr>
<tr>
<td>momentum</td>
<td>234.262</td>
<td>232.462336</td>
<td>0.8%</td>
<td>0.0%</td>
</tr>
<tr>
<td>protol</td>
<td>170195</td>
<td>170175</td>
<td>0.6%</td>
<td>0.0%</td>
</tr>
<tr>
<td>liu</td>
<td>1138</td>
<td>1133.1726</td>
<td>0.5%</td>
<td>0.0%</td>
</tr>
<tr>
<td>dano3mip</td>
<td>687.733333</td>
<td>687.733333</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>


**Xpress-MP Heuristics**

Heuristics become more and more important. Many problems are simply too large and difficult to solve and even explore a small part of the search tree for finding good solutions might be too time consuming. Often users are interested to obtain a "good" solution in short time which can be achieved with heuristics only.

**Heuristic Types**
The Xpress-Optimizer contains a large collection of heuristics like
- diving
- constraint branching
- primal selection
- feasibility pump
- special structure heuristics
- combinations of these heuristics

**Heuristic Improvements**
To document the improvements we made with regard to heuristics, we compare the best solutions found at the root node of the branch-and-bound tree (1% better) with how often a solver found a solution (more than 1% better).

<table>
<thead>
<tr>
<th>% Better</th>
<th>Old</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>25</td>
<td>95</td>
</tr>
<tr>
<td>0%</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>-2%</td>
<td>10</td>
<td>22</td>
</tr>
</tbody>
</table>

out of 130 instances.

**Effectiveness of Heuristics Implemented in Xpress-MP**
To show the effectiveness of the heuristics implemented in Xpress-MP we measure the increase (by geometric mean) in running time up to a certain gap when switching the heuristics off.

<table>
<thead>
<tr>
<th>% Increase</th>
<th>Old</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
<td>20%</td>
<td>2.5</td>
<td>5.0</td>
</tr>
<tr>
<td>30%</td>
<td>3.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Obviously, x is optimal, if

\[
\min \pi^\top x - \pi^\top (Ax - b) \quad \text{s.t.} \quad x \in (0, 1)^n \quad (LR)
\]

So the goal becomes to manipulate \( x \) in a way that not only achieves a good value for (LR) but also an \( x \) which is feasible for the original LP.

Since most problems do not consist of SPPC constraints only, it was necessary to generalize the heuristic so that it can cope with at least one additional constraint type and integer variables.

To make this heuristic work extensive parameter tuning has to be performed.

**Computational Example**

We compare the SPPC heuristic to Xpress-First (which stops after Xpress-Optimizer has found the first solution) and Xpress-Root (which stops after the root node has been processed).

There are two variants of the SPPC heuristic: SPPC-Fast and SPPC-Quality.

The comparison is based on 92 problems.

**Xpress-MP**

Xpress-MP is a suite of mathematical modeling and optimization tools used to solve linear, integer, quadratic, non-linear, constraint and stochastic programming problems. The Xpress-MP suite is available on all common computing platforms, and can be used on all types of computers for solving problems of various sizes. The products support a range of user-friendly interfaces including scalable library APIs in C, C++, VB, Java, .NET and standalone command-line interfaces.

**Solver Engines**

The Xpress-Optimizer features optimization algorithms which enable you to solve linear programming problems (LP), mixed integer programming problems (MIP), quadratic programming problems (QP) and mixed integer quadratic programming problems (MIQP). The Xpress-MP Optimizer offers primal/dual simplex algorithms, a barrier solver and a network simplex solver for solving linear programs. The barrier solver is available for parallel solving.

Xpress-SP is a solver for non-linear programming problems (NLP) and mixed integer non-linear programming problems (MINLP).

Xpress-SP is a Stochastic Programming tool for solving optimization problems involving uncertainty. Xpress-SP can be used to model and solve problems occurring in Supply Chain Management, Energy, Finance, Transportation, etc.

**Model Building and Development Tools**

Xpress-Mosel allows you to formulate your problem, solve it using one or several of the Xpress solver engines, and analyze the solution, using a fully-featured compiled programming language specifically designed for this purpose. The Xpress-Mosel environment comprises the Mosel language with its debugger, modules and IDEs for accessing other software components and external data sources directly in this language; libraries for embedding models into applications; and an open interface for user-written extensions to the Mosel language.

Xpress-ILOG is an object-oriented library for building, solving, and analyzing problems directly within an application. Xpress-ILOG is a complete visual development environment for Xpress-Mosel under Windows. It incorporates a Mosel program editor, compiler and execution environment.

**Teaching with Xpress-MP**

Dash Optimization has put together a special program for students and educators interested in teaching optimization. Visit our web-page for more details.

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