Xpress-MP performance

Solving Hard Mixed Integer Programs with Xpress-MP

We constantly benchmark our optimization engines on public test sets and on customer instances.

Speed Up on Internal Test Set

The current release 2007A of the Xpress-Optimizer is
- 22% faster by geometric mean and
- 41% faster by arithmetic mean

than 2006A on our internal test set.

Solving Unsolved Instances from MIPLIB 2003

The MIPLIB 2003 is a collection of 60 MIPs of which many are hard-to-solve or remain unsolved until today.
With the latest improvements in Xpress-Optimizer, it was possible to
- solve 10 (out of 17) problems which where unsolved before Dec-2005.
- solve 5 new problems for the 1st time: atlanta-ip, msc98-ip, protfold, rd-rplusc-21 and sp97ar.
- solve a1c1s1 for the first time on a single computer
- find the 1st feasible solution for the stp3d problem (500.736 solution which is within 3.3% of the optimum).
- find better solutions for all current 6 unsolved problems

Main MIP Components that Lead to the MIPLIB 2003 Improvements

The following improvements were crucial to obtain these results.
- presolving
  - node-to-node preprocessing
- cutting
  - use of lift-and-project cuts
- better branching decisions
  - improved strong branching estimates

Besides these, for some instances it was useful to search with a special strategy for a good solution and use this to restrict the search for the bound raising tree.

To be able to raise the bound faster, we explored the top of the tree with heavy strong branching for a certain number of nodes, applied a full presolve on the leaf nodes of this tree and continued the search from this point on.

The results in more detail are the following:

Optimal Solutions

Problem	Old Best Known Obj. Value	Xpress Improved Obj. Value	GAIN
atlanta-ip	95.009549704	90.00987861	5.3%
msc98-ip	20980991.006	19839497.006	5.4%
protfold	-30	-31	3.3%
rd-rplusc-21	171182	165395.2753	3.5%
sp97ar	664565103.76	660705646.8	0.6%

New Primal and Dual Bounds

Problem	Old Best Known Obj. Value	Xpress Improved Obj. Value	GAIN
ds	283.4425	116.59	58.9%
momentum3	370177.036	236426.335	36.1%
t1717	193221.036	170195	11.9%
liu	1172	1138	2.9%
dano3mip	691.2	687.733333	0.5%

Richard Laundy, Michael Perregaard, Gabriel Tavares, Horia Tipi and Alkis Vazacopoulos, *Solving HardMixed Integer Programming Problems with Xpress-MP: A MIPLIB 2003 Case Study*, RRR 2-2007, Rutgers University, 2007.

Xpress-MP Heuristics

Heuristics become more and more important. Many problems are simply too large and difficult to solve and even exploring a small part of the search tree for finding good solutions might be too time consuming. Often users are interested to obtain a “good” solution in short time which can be achieved with heuristics only.

Heuristic Types

The Xpress-Optimizer contains a large collection of heuristics like
- diving
- constraint branching
- local search
- feasibility pump
- special structure heuristics
- combinations of these heuristics

Heuristic Improvements

To document the improvements we made with regard to heuristics, we compare the best solutions found at the root for different solvers. The column “$>1\%$ better” denotes how often a solver found a (more than 1%) better solution.

2007A vs. 2005B	$>1\%$ better	only solver to find a solution
2005B is best	23	2
2007A is best	78	9

out of 130 instances.

Effectiveness of Heuristics Implemented in Xpress-MP

To show the effectiveness of the heuristics implemented in Xpress-MP, we measure the increase (by geometric mean) in running time up to a certain gap when switching the heuristics off.

Increase of running time without heuristics	
Time to first solution	+145%
Time to 5% gap	+43%
Time to 0.5% gap	+20%
Time to optimality (0.01% gap)	+6%

Non LP-Based Heuristics

Most MIP heuristics need the LP solution to start with or solve iteratively LPs to find an integer feasible solution.
There are two drawbacks of this approach
- the heuristics have to wait for the LP relaxation to be solved.
- the LP solve times might be long on their own.

Since parallel processing becomes more and more available, the time needed for solving the initial LP relaxation might be used to find heuristic solutions if the heuristic runs independent of an LP solution/solver.

A Packing and Covering Heuristic

One of these heuristics has been suggested by Dag Wedelin in the context of airline crew scheduling. It works best on set packing, partitioning and covering (SPPC) problems.
The basic idea is to consider the problem’s Lagrangian relaxation:

$$\begin{aligned} &\min\ c^\top x - \pi^\top (Ax - b) \\ &\text{s.t. } x \in \{0,1\}^n \end{aligned} \qquad \text{(LR)}$$

where π constitutes the *Lagrangian multipliers.* It is trivial to find the minimum for the problem above once the value of π has been fixed as can be seen using the following equivalent formulation:

$$\begin{aligned} &\pi^\top b + \min\ (c^\top - \pi^\top A)x \\ &\text{s.t. } x \in \{0,1\}^n \end{aligned}$$

Obviously x is optimal, iff

$$x_i = \begin{cases} 1 & \text{if } (c^\top - \pi^\top A)_i < 0 \\ 0 \text{ or } 1 & \text{if } (c^\top - \pi^\top A)_i = 0 \\ 0 & \text{if } (c^\top - \pi^\top A)_i > 0. \end{cases}$$

So the goal becomes to manipulate π in a way that not only achieves a good value for (LR) but also an x which is feasible for the original IP.
Since most problems do not consist of SPPC constraints only, it was necessary to generalize the heuristic so that it can cope with at least a few additional constraint types and integer variables.
To make this heuristic work extensive parameter tuning has to be performed.

Computationals

We compare the SPPC heuristic to Xpress-First (which stops after Xpress-Optimizer has found the first solution) and Xpress-Root (which stops after the root node has been processed). There are two variants of the SPPC heuristic, SPPC-Fast and SPPC-Quality.

		X-F	X-R	S-F	S-Q
Solver	time	is better than			
X-F	3933	-	63	29	31
X-R	5437	0	-	10	17
S-F	573	39	52	-	43
S-Q	2609	40	45	12	-
Xpress-LP	1705				

The comparison is based on 92 problems.

Oliver Bastert, Benjamin Hummel and Sven de Vries, *A Generalized Wedelin Heuristic for Integer Programming*, submitted.

Xpress-MP

Xpress-MP is a suite of mathematical modeling and optimization tools used to solve linear, integer, quadratic, nonlinear, constraint and stochastic programming problems.
The Xpress-MP suite is available on all common computer platforms and in different capacities for solving problems of various sizes. The products support a range of user/software interfaces including callable library APIs in C, C++, VB, Java, .NET and standalone command line interfaces.

Solver Engines

The Xpress-Optimizer features optimization algorithms which enable you to solve linear programming problems (LP), mixed integer programming problems (MIP), quadratic programming problems (QP) and mixed integer quadratic programming problems (MIQP). The Xpress-Optimizer offers primal/dual simplex algorithms, a barrier solver and a network simplex solver for solving linear programs. The barrier and the MIP solver offer parallel solving.
Xpress-SLP is a solver for non-linear programming problems (NLP) and mixed integer non-linear programming problems (MINLP).
Xpress-SP is a Stochastic Programming tool for solving optimization problems involving uncertainty. Xpress-SP can be used to model and solve problems occurring in Supply Chain Management, Energy, Finance, Transportation, etc.
Xpress-Kalis is Constraint Programming software based upon the Kalis solver by Artelys.

Model Building and Development Tools

Xpress-Mosel allows you to formulate your problem, solve it using one or several of the Xpress solver engines, and analyze the solution, using a fully-functional compiled programming language specifically designed for the purpose.
The Xpress-Mosel environment comprises the Mosel language with its debugger; modules and I/O drivers for accessing other software components and external data sources directly in this language; libraries for embedding models into applications; and an open interface for user-written extensions to the Mosel language.
Xpress-BCL is an object-oriented library for building, solving, and analyzing problems directly within an application.
Xpress-IVE is a complete visual development environment for Xpress-Mosel under Windows. It incorporates a Mosel program editor, compiler and execution environment.

Teaching with Xpress-MP

Dash Optimization has put together a special program (Academic Partner Program (APP)) for degree awarding academic institutions where academics and their students may use Xpress-MP for educational purposes. In addition, academics may use Xpress-MP for research and consulting activities.
Visit our web-page for more details.

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