Non-commutative twisting in motivic and étale cohomology

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Abstract

This is joint work with Guido Kings. We prove that under a technical condition (i.e. the Iwasawa $\mu = 0$ conjecture), for any smooth projective scheme X over a number ring $O_K[1/S]$, the étale cohomology groups $H^1\left(O_K[1/S], H^i(\overline{X}, \mathbf{Q}_p(j)\right)$ are generated by twists of norm compatible units in a non-abelian tower of number fields with elements of $H^i\left(\overline{X}, \mathbf{Z}_p(j-1)\right)$ if $j \ll 0$. Assuming the "Bloch–Kato-conjecture", we establish a similar result for motivic cohomology with finite coefficients. Recall that $\mu = 0$ is known if the extension $K/\mathbf{Q}$ is abelian, and that conjecturally

$$H^1\left(O_K[1/S], H^i\left(\overline{X}, \mathbf{Q}_p(j)\right)\right) \cong H^{i+1}_{\mathrm{mot}}\left(X, \mathbf{Z}(j)\right) \otimes_{\mathbf{Z}} \mathbf{Q}_p \quad \text{if } j > i+1.$$