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Non-logarithmically concave Littlewood-Richardson coefficients

Jerzy Weyman
j.weyman@neu.edu
Department of Mathematics
Northeastern University
360 Huntington Avenue
Boston, MA 02115
USA

Abstract

I will explain how one can use quiver representations to prove some results on Littlewood-Richardson coefficients. Recall that for three partitions λ, μ, ν the Littlewood-Richardson coefficient $c^{\nu}_{\lambda,\mu}$ describes the multiplicity of the Schur module $S_\nu(\mathbf{C}^n)$ in the tensor product $S_\lambda(\mathbf{C}^n) \otimes S_\mu(\mathbf{C}^n)$. For three partitions λ, μ and ν it is natural to consider the function $f(n) := c^{n\nu}_{n\lambda,n\mu}$. Such functions were conjectured by Okunkov to be logarithmically concave, i.e. to satisfy the inequalities $f(n-1)f(n+1) \leq f(n)^2$ for every n. The conjecture would immediately imply the Saturation Theorem of Knutson and Tao. We show, however, that the conjecture fails. The use of quiver representations is essential to find the triple λ, μ, ν for which conjecture fails. I will also preseent some results and other conjectures regarding the functions $f(n)$, and, more generally the weight multiplicities of rings of semi-invariants of quiver representations.

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