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Entanglement entropy in quantum spin chains with finite range interaction

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Abstract

We study the entropy of entanglement of the ground state in a wide family of one-dimensional quantum spin chains whose interaction is of finite range and translation invariant. Such systems can be thought of as generalizations of the XY model. The chain is divided in two parts: one containing the first consecutive L spins; the second the remaining ones. In this setting the entropy of entanglement is the von Neumann entropy of either part. At the core of our computation is the explicit evaluation of the leading order term as L tends to infinity of the determinant of a block-Toeplitz matrix whose symbol belongs to a general class of 2×2 matrix functions. The asymptotics of such determinant is computed in terms of multi-dimensional theta-functions associated to a hyperelliptic curve of genus $g >= 1$, which enter into the solution of a Riemann–Hilbert problem. Phase transitions for thes systems are characterized by the branch points of the hyperelliptic curve approaching the unit circle. In these circumstances the entropy diverges logarithmically. We also recover, as particular cases, the formulae for the entropy discovered by Jin and Korepin (2004) for the XX model and Its, Jin and Korepin (2005, 2006) for the XY model.