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## Quantum geometry of 3-dimensional lattices

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### Abstract

We study geometric consistency relations between angles on 3-dimensional (3D) circular quadrilateral lattices—lattices whose faces are planar quadrilaterals inscribable into a circle. We show that these relations generate canonical transformations of a remarkable “ultra-local” Poisson bracket algebra defined on discrete 2D surfaces consisting of circular quadrilaterals. Quantization of this structure leads to new solutions of the tetrahedron equation (the 3D analog of the Yang–Baxter equation). These solutions generate an infinite number of non-trivial solutions of the Yang–Baxter equation and also define integrable 3D models of statistical mechanics and quantum field theory. The latter can be thought of as describing quantum fluctuations of lattice geometry. The classical geometry of the 3D circular lattices arises as a stationary configuration giving the leading contribution to the partition function in the quasi-classical limit.