

The geometry of holomorphic and algebraic curves in
complex algebraic varieties
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Weak approximation and rational simple connectedness

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Abstract

Given a system of homogeneous polynomial equations in some variables and depending on some parameters, C. Tseng and S. Lang proved there exist solutions which are rational functions of the parameters, provided the degrees of the polynomials, the number of variables, and the number of parameters satisfy a suitable inequality.

Stronger than existence of rational solutions is “weak approximation”: the ability to approximate any power series solution arbitrarily well by rational solutions. A beautiful, simple argument of B. Hassett relates weak approximation for one-parameter systems to the property of “rational simple connectedness”—roughly, rational connectedness of spaces of rational curves in a general fiber. Recently de Jong and I proved all smooth complete intersections of sufficiently small degree are rationally simply connected. In particular, a one-parameter family of degree d hypersurfaces in projective n -space whose general member is smooth satisfies weak approximation if

$$n > 2d^2 - d - 2.$$

The proof works by deducing rational simple connectedness from positivity of the Chern character using uniruledness results of Kollár, Miyaoka and Mori.