Logarithmic surfaces and hyperbolicity

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Abstract

In 1981 J. Noguchi proved that in a logarithmic algebraic manifold, having logarithmic irregularity strictly bigger than its dimension, any entire curve is algebraically degenerate.

In the present paper we are interested in the case of manifolds having logarithmic irregularity equal to its dimension. We restrict our attention to Brody curves, for which we resolve the problem completely in dimension 2:

Theorem. *In a logarithmic surface with logarithmic irregularity 2 and logarithmic Kodaira dimension 2, any Brody curve is algebraically degenerate.*

In the case of logarithmic Kodaira dimension 1, we still get the same result under a very mild condition on the Stein factorization map of the quasi-Albanese map of the log surface, but we show by giving a counter-example that the result is not true any more in general. Finally we prove that a logarithmic surface having logarithmic irregularity 2 admits certain types of algebraically non degenerate entire curves iff its logarithmic Kodaira dimension is zero, and we also give a characterization of this case in terms of the quasi-Albanese map. As a corollary, we get hyperbolicity for such logarithmic surfaces not containing non-hyperbolic algebraic curves and having hyperbolically stratified boundary divisors. In particular we get the "best possible" result on algebraic degeneracy of Brody curves in the complex plane minus a curve consisting of three components, thus improving results of Dethloff-Schumacher-Wong from 1995.