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Nevanlinna-Galois theory for the Gauss map of algebraic minimal surfaces

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Abstract

Let M be an algebraic minimal surface and $g\colon \mathbb{D} \to \mathbb{P}^1$ the Gauss map lifted to its universal cover $\mathbb{D}$. The Nevanlinna height $T_g(r)$ of g (i.e., the height transform of the Gauss area function $(0,1) \ni t \mapsto \mathrm{Area}_{g^*\omega_{\mathrm{FS}}}\big(\mathbb{D}(t)\big) \in \mathbb{R}_+$) and the height transform $T_{\mathrm{hyp}}(r)$ of the area function $(0,1) \ni t \mapsto \mathrm{Atra}_{\mathrm{hyp}}\big(\mathbb{D}(t)\big) \in \mathbb{R}_+$ are of fundamental objects in the study of the Gauss map of algebraic minimal surfaces. The comparison of these two heights should be based on the Nevanlinna Theory on the disk $\mathbb{D}$ under the action of the fundamrental group $\pi_1(M)$. In this situation, we establish the following two results. The first is an inequality of the form $T_g(r) \geq (\mathrm{const.})\, T_{\mathrm{hyp}}(r)$, which is interpreted as a Nevanlinna-Galois version of the Cohn-Vossen inequality. The second is a Nevanlinna-Galois expression of the period condition of the corresponding algebraic minimal surface (this is a formula which expresses the height $T_g(r)$ in terms of some Nevanlinna theoretic functions constructed only from the action of $\pi_1(M)$ on $\mathbb{D}$). As a result we show a version of the Second Main Theorem for the (lifted) Gauss map of algebraic minimal surfaces.