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Operator Hölder–Zygmund functions

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I am going to speak about my joint results with A.B. Aleksandrov. It is well known that a Lipschitz function is not necessarily operator Lipschitz which means that for a Lipschitz function f on the real line it is not necessarily true that

$$\|f(A) - f(B)\| \leq \text{const} \|A - B\|$$

for self-adjoint operators A and B . It is also well known that a continuously differentiable function is not necessarily operator differentiable. However, we have proved that if f is a function in the Hölder class Λ_α with $0 < \alpha < 1$, then it is operator Hölder, i.e.,

$$\|f(A) - f(B)\| \leq \text{const} \|A - B\|^\alpha$$

for self-adjoint operators A and B . The same is true for functions in the Zygmund class Λ_1 . They must be operator Zygmund, i.e.,

$$\|f(A + K) - f(A) + f(A - K)\| \leq \text{const} \|K\|$$

for self-adjoint operators A and K . The same is true for all spaces Λ_α , $0 < \alpha < \infty$, of the Hölder–Zygmund scale. Similar results also hold for functions of unitary operators and for functions of contractions.