

CONFÉRENCE « ESPACES DE HILBERT DE FONCTIONS ANALYTIQUES »
8–12 DÉCEMBRE 2008

CONFERENCE ON HILBERT SPACES OF ANALYTIC FUNCTIONS
DECEMBER 8–12, 2008

On the asymptotic zero distribution of some planar orthogonal polynomials

Ferenc Balogh

Department of Mathematics & Statistics
Concordia University
1455 de Maisonneuve Blvd. W.
Montréal, QC H3G 1M8
CANADA

fbalogh@mathstat.concordia.ca

It is a well-known fact in the theory of random normal matrices that the so-called correlation functions of the eigenvalues can be expressed in terms of certain orthogonal polynomials. The asymptotic properties of the statistical quantities as the matrix size goes to infinity are therefore encoded into those of the orthogonal polynomials in the large degree limit. We consider external fields given by potential functions of the form $V(z) = |z|^2 + h(z)$ where h is sub- or superharmonic function and harmonic on a subdomain of the complex plane, dominated by the Gaussian term at infinity (in most cases h given by the logarithmic potential of a compactly supported finite measure in the plane). Motivated by matrix models, a one-parameter family of orthogonal polynomials $\{P_{n,N}(z)\}_n$ is constructed with respect to the varying weights of the form $\exp(-NV(z))$ depending on a positive parameter N . The talk is focused on the limiting behavior of the zeroes of such orthogonal polynomials $P_{n,N}(z)$ in the scaling limit $n \rightarrow \infty$, $N \rightarrow \infty$, $N/n \rightarrow \gamma > 0$. A general conjecture on the asymptotic distribution of the zeroes will be presented, supported by some nontrivial special cases and numerical calculations.

The talk is based on joint works with M. Bertola, J. Harnad, S.Y. Lee, and K. McLaughlin.