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de Branges–Rovnyak reproducing kernel spaces : multivariable extensions

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The now classical de Branges–Rovnyak space is the reproducing-kernel Hilbert space $H(K_S)$ of vector-valued analytic functions on the unit disk associated with the de Branges–Rovnyak kernel $K_S(z,w) = [I - S(z)S(w)*]/(1 - z\overline{w})$ where S is a Schur-class operator-valued function (holomorphic with contractive operator values on the unit disk). It is known that any Schur-class function can be realized in the form of the transfer function of a linear system $S(z) = D + zC(I - zA)^{-1}B$ where the block 2-by-2 colligation matrix $[A\ B//C\ D]$ is coisometric from the $H \oplus U$ (the state-space direct sum the input space) to $H \oplus Y$ (the state-space direct sum the output space). Moreover one can take the state space H to be equal to the reproducing-kernel space $H(K_S)$ and the operator A equal to the backward shift restricted to $H(K_S)$. We discuss extensions of these results to various multivariable settings where the unit disk is replaced by (1) the ball in d-dimensional complex Euclidean space, (2) the noncommutative d-variable operator ball in $L(K)^d$ (K a fixed infinite-dimensional separable Hilbert space), (3) the polydisk in d-dimensional complex Euclidean space, and (4) the noncommutative polydisk in $L(K)^d$.

The talk reports on joint work with a number of collaborators (*Victor Vinnikov, Cora Sadosky, Dima Kaliuzhnyi–Verbovetskyi, Vladimir Bolotnikov and Quanlei Fang*).