Large Precomputation methods and the Evans function

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Abstract

We study the problem of sampling the solution to a system of linear non-autonomous ordinary differential equations which depends linearly on a set of parameters across parameter space. If we use a standard numerical method, we must re-integrate the solution at every sampling point. This is particularly inefficient when high accuracy over a widespread set of parameter values is required. Continuity methods can resolve this issue locally, but still require some degree of re-integration. We present a new efficient numerical method that absolves us of re-integration completely. The basic idea is to first expand either the Neumann or Magnus series solution of our system as a power series in the parameters. The coefficients of the series expansion can be precomputed to any given accuracy. We can then simply evaluate the series for each parameter value we wish to sample.

Systems of this type arise when using the Evans function method to determine the linear stability of travelling wave solutions to reaction-diffusion and related partial differential equations. We demonstrate the effectiveness of our method by computing the well-known onset of the 'pulsating instability' for travelling waves in a model of autocatalysis.

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