

Conservation laws of semidiscrete canonical Hamiltonian equations

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Abstract

There are many evolution partial differential equations which can be cast into Hamiltonian form. Conservation laws of these equations are related to one-parameter Hamiltonian symmetries admitted by the PDEs. The same result holds for semidiscrete Hamiltonian equations. In this presentation we consider semidiscretizations of canonical Hamiltonian equations

$$u_t = \frac{\delta\mathcal{H}}{\delta w}, \quad w_t = -\frac{\delta\mathcal{H}}{\delta u}.$$

Due to their special structure these equations possess conservation of symplecticity. Using symmetries, we find conservation laws for the semidiscretized nonlinear wave equation

$$v_{tt} = v_{xx} - V'(v),$$

and the nonlinear Schrödinger equation

$$i\psi_t + \psi_{xx} + F'(|\psi|^2)\psi = 0$$