Continuum Limit of Lattice Approximation Schemes

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Abstract

Boundary-layer perturbation theory problems are inherently singular. However, it is known that discretizing the problem by introducing a lattice may convert such problems into regular perturbation problems. The singular nature of boundary-layer problems is then relegated to and hidden in the continuum limit, the subtle limit in which the lattice spacing tends to zero. If the lattice is introduced cavalierly, then extrapolating to zero lattice spacing gives a sequence of extrapolants that at first approach the correct limit and then veer off, thereby revealing the asymptotic nature of such problems. However, discretizing the problem following the procedures described here yields lattice approximations that have a smooth and regular continuum limit. These ideas are illustrated by three nonlinear ordinary differential equations: the cubic equation that describes instantons, an oscillator equation having a quadratic nonlinearity, and the Blasius equation.