Conservation laws of the Teukolsky equation for massless spin s fields in Kerr spacetime

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Abstract

The Teukolsky equation describes a linear, separable wave equation for propagation of massless fields of spin $s = 0, 1/2, 1, 3/2, 2, ...$ in Kerr spacetime, using a complex scalar field representing the components of spin-weight $\pm s$ with respect to a null tetrad in the Newman-Penrose formalism. This field equation, however, has no direct Lagrangian formulation for $s \neq 0$, which has presented an obstacle to obtaining local conservation laws for energy and angular momentum carried by spin-weight scalar fields in Kerr spacetime. Such conserved quantities may be expected to be important for obtaining estimates on long-time decay behavior of electromagnetic ($s = 1$) and gravitational ($s = 2$) perturbations of Kerr blackholes, generalizing recent work in the case of scalar perturbations ($s = 0$) by Finster, Kamran, Smoller, and Yau. Here we derive energy and angular momentum conservation laws of spin-weight scalar fields for all spins $s \geq 0$ from the Teukolsky equation by means of a generalization of Noether's theorem employing adjoint symmetries, combined with radial and angular operator techniques introduced by Chandrasekhar. We discuss various features (positivity, asymptotic form, flat-spacetime limit, relation to stress-energy conservation laws, etc.) of the resulting spin s energy and angular momentum expressions.