Szego limit theorem for operators with discontinuous symbols acting in R^d

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Abstract

Let each of I and J be a finite union of bounded intervals. Let A be a pseudodifferential operator with symbol $\sigma(x, \xi)$. In 1981, H. Widom established a two-term asymptotic formula, as $\lambda \to +\infty$, for the trace of an analytic function of the operator of restriction to I followed by the operator of restriction by λJ on the Fourier transform side, followed by A and finally by the restriction to I again. Also a two-term formula for the higher dimensional case was conjectured in this paper.

This work was preceded by a paper by Widom and H.J.Landau for $\sigma = 1$. This problem is related to the general question that arises for example in signal processing: What can we say about a function if we know its restriction to the set I and the restriction of its Fourier transform to the set J?

In 1990, Widom settled the conjecture for the higher dimensional case under the assumption that one of the sets is compact with smooth boundary and the other is a half-space. It was possible to effectively reduce this problem to the one dimensional case.

We present a partial result towards Widom's conjecture. This result is a one-term asymptotic formula with a sharp remainder estimate which holds for compact sets I and J in R^d that satisfy mild regularity conditions. If the boundaries are C^1, we get the leading term in

Widom's asymptotics and the remainder that has the order that the conjectured second term should have.

We will explain the difficulty that arises if one tries to complete the proof of Widom's conjecture. Also we discuss certain inequalities between tail integrals of Fourier transforms and general moduli of continuity in the spirit of D.B.H. Cline's 1991 paper.