

## Simplicial nonpositive curvature

Jacek Swiatkowski  
swiatkow@math.uni.wroc.pl  
*Mathematical Institute*  
*Wroclaw University*  
*pl. Grunwaldzki 2/4*  
*50-384 Wroclaw*  
*POLAND*

### Abstract

Simplicial nonpositive curvature is a purely combinatorial notion (applicable to simplicial complexes) which resembles metric nonpositive curvature. It has been introduced recently in my joint work with Tadeusz Januszkiewicz (and some aspects independently by F. Haglund). Having no direct relationship with metric nonpositive curvature, the notion has numerous similar consequences. For example, simplicially nonpositively curved complexes are aspherical, their fundamental groups are biautomatic (and thus semi-hyperbolic), and simplicially nonpositively curved complexes of groups are developable. Moreover, a slightly stronger variant of the concept yields Gromov-hyperbolicity.

Developability of simplicially nonpositively curved complexes of groups allows construction of examples, also in higher dimensions. This leads to solutions of open problems concerning existence of:

- hyperbolic Coxeter groups with arbitrary  $vcd$ ,
- CAT(0) developments of simplicial billiard tables of any dimension,
- simple criterion for Gromov-hyperbolicity of simplicial complexes of arbitrary dimension, and many others.

Simplicial nonpositive curvature does not fit well to manifolds of dimension above 2. The corresponding spaces and groups in dimensions above 2 turn out to have rather exotic properties. Their further study seems worthwhile, as well as looking for further applications and generalizations.

In the mini-course I am going to introduce the concept of simplicial nonpositive curvature, prove its basic properties and consequences, describe construction of examples, discuss applications and show some exotic properties in high dimensions.