

L^2 -Betti numbers

Michael W. Davis
mdavis@math.ohio-state.edu
Department of Mathematics
Ohio State University
231 W. 18th Ave.
Columbus, OH 43210
USA

Abstract

Suppose a discrete group Γ acts properly on a CW complex X with compact quotient space. One then defines the (reduced) L^2 -cohomology of X . It is a Hilbert space with an orthogonal Γ -action. When these Hilbert spaces are nonzero they tend to be infinite dimensional. However, by using the Γ action, one can define their “ Γ -dimension” (or “von Neumann dimension”). This is a nonnegative real number. It is 0 if and only if the corresponding Hilbert space is 0. When applied to the L^2 -cohomology of X , Γ -dimension yields the L^2 -Betti numbers. I will explain the two pre-eminent conjectures in this area, the Atiyah Conjecture and the Singer Conjecture. I’ll discuss the result of Cheeger–Gromov that if Γ is amenable and X is contractible, then all its L^2 -Betti numbers vanish. Finally I’ll discuss the new theory of “weighted L^2 cohomology” which is applicable in the case when Γ is a Coxeter group.