

# Surface groups, $\mathrm{SL}(n, \mathbb{R})$ , crossratios and $C^1(S^1) \rtimes \mathrm{Diff}^1(S^1)$

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## **Abstract**

In this talk, I plan to explain the geometric significance of a connected component of the space of representations of a surface group in  $\mathrm{SL}(n, \mathbb{R})$  as the symmetries of some special curves in projective spaces. Then, I explain how this gives a description of these spaces as space of “crossratios” on the circle, namely functions of four points satisfying some functional relations. Finally, I will explain that if  $n$  goes to infinity all the corresponding moduli spaces sit in a unique moduli space which can be interpreted as the space of representations of the surface group in  $C^1(S^1) \rtimes \mathrm{Diff}^1(S^1)$ , which one may grossly think of as a central extension of a subgroup of Hamiltonian diffeomorphisms of the annulus. This latter space also contains the space of all negatively curved metrics on the surface. If time permits, I will also state a conjecture that would give the holomorphic version of this description. The general idea being that the results explained here are a generalisation of Teichmüller theory.