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*Moduli spaces of self-dual connections over
asymptotically locally flat gravitational
instantons*

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Abstract

We investigate Yang–Mills instanton theory over four dimensional asymptotically locally flat (ALF) geometries, including gravitational instantons of this type, by exploiting the existence of a natural smooth compactification of these spaces introduced by Hausel–Hunsicker–Mazzeo.

First referring to the codimension 2 singularity removal theorem of Sibner–Sibner and Rade we prove that given a smooth, finite energy, self-dual $SU(2)$ connection over a complete ALF space, its energy is congruent to a Chern–Simons invariant of the boundary three-manifold if the connection satisfies a certain holonomy condition at infinity and its curvature decays rapidly.

Then we introduce framed moduli spaces of self-dual connections over Ricci flat ALF spaces. We prove that the moduli space of smooth, irreducible, rapidly decaying self-dual connections obeying the holonomy condition with fixed finite energy and prescribed asymptotic behaviour on a fixed bundle is a finite dimensional manifold. We calculate its dimension by a variant of the Gromov–Lawson relative index theorem.

As an application, we study Yang–Mills instantons over the flat $R^3 \times S^1$ (also called calorons), the multi-Taub–NUT family, and the Riemannian Schwarzschild space.

This is a joint work with M. Jardim.