Real zeros of Rankin-Selberg L-functions in the level aspect

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Abstract

In 2002, J.B. Conrey and K. Soundararajan have shown that there are infinitely many Dirichlet L-functions which do not vanish on the critical segment. Two crucial remarks about their result:

- the analytic technique used (mollification method) and their zeros counting method compell them to get strong asymptotic formulae of the mollified second moment of their family $\mathcal{G}$ of Dirichlet L-functions at a distance of the inverse of the log of the analytic conductor of their family,
- all their work is justified by the random matrix model of their family $\mathcal{G}$: its symmetry type is the symplectic one which entails a repulsion of the first zero a little away from the critical segment.

Following their work, a similar analytic study (especially the non-trivial real zeros but also the size on the critical line) of a family $\mathcal{F}$ of Rankin-Selberg L-functions which has the same symmetry type has been undertaken. Some strong asymptotic formulae for the harmonic mollified second moment of $\mathcal{F}$ have been established and obviously the asymptotic of the harmonic mollified second moment of $\mathcal{F}$ is the same than the asymptotic of the mollified second moment of $\mathcal{G}$. The main contribution is a substancial improvement of the admissible length of the mollifier which is done by solving a shifted convolution problem

by a spectral method of P. Sarnak on average thanks to large sieve inequalities for Fourier coefficients of Maass forms. Some other refinements obtained by B. Krtz and R.J. Stanton are also needed.