Atelier ≪ Équations et propriétés du premier ordre dans les groupes ≫
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Algebraic geometry over rigid groups

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If G is a solvable group of class m then there is a series of normal subgroups

$$G = G_1 > G_2 > \cdots > G_m > G_{m+1} = 1$$

with abelian factors G_i/G_{i+1}. The action of G by conjugation $x \to g^{-1}xg$ defines on G_i/G_{i+1} a structure of a $\mathbb{Z}[G/G_i]$-module. The group G is said to be rigid (m-rigid) if G_i/G_{i+1} have no torsion as modules. For example, free solvable groups are rigid. We prove that rigid groups are equationally noetherian, which means that any system of equations in finitely many variables with coefficients in a given group is equivalent to some finite subsystem. This fact makes it possible to develop an algebraic geometry over rigid groups. We prove that an arbitrary m-rigid group can be embedded into some divisible (such that all factors G_i/G_{i+1} are divisible modules) m-rigid decomposed group and describe the coordinate groups of irreducible algebraic sets over the latter group.

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