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Nullstellensatz for algebraic structures

Vladimir N. Remeslennikov *

remesl@ofim.oscsbras.ru

Classical Hilbert’s theorem, known as the Nullstellensatz, was originally proven in 1893 for algebraically closed fields. This theorem shows how to compute the radical $\text{Rad}(S)$ of a system of equations S over an algebraically closed field k : a polynomial f belongs to $\text{Rad}(S)$ if and only if some power of f belongs to the ideal generated by S .

In this talk I will first discuss computation of radicals of systems of equations over real numbers and over p -adic rationals. The notions of equation and radical can be given for an arbitrary algebraic structure A . I will then discuss the problem of computing the radical for arbitrary system of equations S over A .

In the general case we use the term “Nullstellensatz” for theorems that give a description or a procedure for computing the radical. I will discuss the Nullstellensatz for torsion-free abelian and free metabelian groups.

*Department of Mathematics., Omsk State University, Prospect Mira, 55-A Omsk 644077, Russia.