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Existential and positive theories of graph products over groups

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Consider a finite undirected graph (V, E) such that for each vertex $v \in V$ there is an associated group G_v. Then the graph product is the free product over these G_v modulo defining commutation relations $ab = ba$ for all $a \in G_u$ and $b \in G_v$ such that $uv \in E$. Thus e.g., if the graph is the complete graph, then the resulting graph product is the direct product $\Pi_{v \in V} G_v$, if on the other extreme, $E = \emptyset$, then the resulting graph product is a free product. If all groups G_v are free, then we obtain a right-angled Artin group, also known as a graph group.

Together with Markus Lohrey we have shown the following results : The existential and positive theories of a graph product can be reduced to the connected components of the underlying dependence graph which is the complement graph to (V, E). For connected dependence graphs it is possible to reduce the positive theory to the existential one by a generalization of the techniques introduced by Merzlyakov for free groups. Decidability of existential theory can be reduced to the decidability of existential theory of each G_v via a techniques which involves free partially commutative monoids with involution and normalized rational constraints.

As a corollary of our results we see that, e.g., the positive theory of graph products over hyperbolic groups is decidable.

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