# Universal Completions of Groups

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Equations and first-order properties in groups

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# Introduction

- Algebraic extensions and algebraically closed field.
- Free groups and the Lyndon's completion.
- Given an arbitrary group $G$, does there exist a "universal" object for the class of f.g. fully residually $G$ groups?

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# Basic definitions

Let $G$ be a group, $F(X)$ be a free group with basis $X = \{x_1, \ldots, x_k\}$. Set $G[X] = G * F(X)$.

- An equation over $G$ is an expression of the form

$$w = 1, \quad \text{where} \quad w \in G * F(X).$$

- A system of equations $S$ over $G$ is a collection of equations.
- A solution of $S$:

$$(g_1, \ldots, g_k) \in G^k \;\text{ so that }\; w(g_1, \ldots, g_k) = 1 \;\text{ in }\; G$$

  for all $w$ in $S$.
- The variety $V(S)$ defined by $S$ is the set of all solutions of $S$.
- Equivalently, a solution of $S$ is a homomorphism

$$\varphi : G[X] \to G \;\text{ so that }\; S \subseteq \ker(\varphi)$$

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# Basic definitions

- A coordinate group of $S$ is:

$$G[X]/_{R(S)} = G[X] \Big/ \bigcap_{\varphi \text{ solution}} \ker(\varphi)$$

- Solutions of $S$ are homomorphisms from $G_{R(S)}$ to $G$;
- The category of coordinate groups is dual to the category of varieties;
- Zariski topology: varieties are pre-basis for closed sets;
- A variety $Y$ is called irreducible if $Y \subseteq Y_1 \cup Y_2$, then $Y \subseteq Y_1$ or $Y \subseteq Y_2$, for any closed sets $Y_1, Y_2$.

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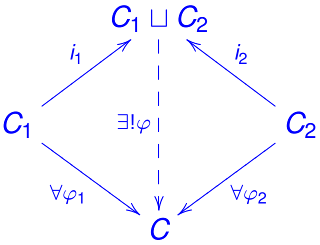
# Unification theorem for equationally Noetherian groups

| | | |
|---|---|---|
| Irreducible variety | $\longleftrightarrow$ | Coordinate group of an irreducible variety |
| $\updownarrow$ | | $\Updownarrow$ |
| f.g. group $\equiv_\forall G$ | $\Longleftrightarrow$ | f.g. fully residually $G$ |
| $\Updownarrow$ | | $\parallel$ |
| Limit groups over $G$ | $\Longleftrightarrow$ | f.g. groups discriminated by $G$ |

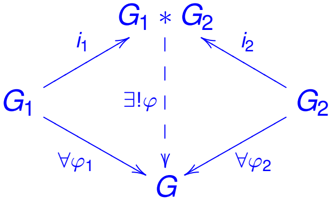
# Coproduct

Let $\mathcal{C} = (\mathrm{Obj}, \mathrm{Hom})$ be a category. Given $C_1, C_2 \in \mathrm{Obj}$ the coproduct $C_1 \sqcup C_2$, if exists, is an element of Obj together with a pair of homomorphism $i_1$ and $i_2$ so that the following diagram commutes:



# Examples

- In the category of groups $\mathcal{G} = (\textit{groups}, \textit{homomorphisms})$, the coproduct corresponds to the free product.

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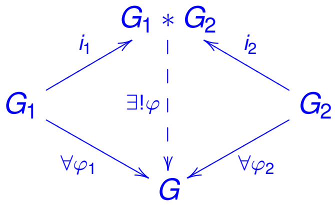


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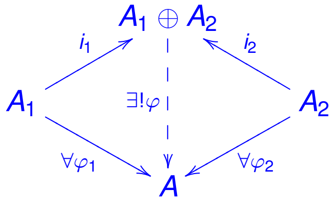
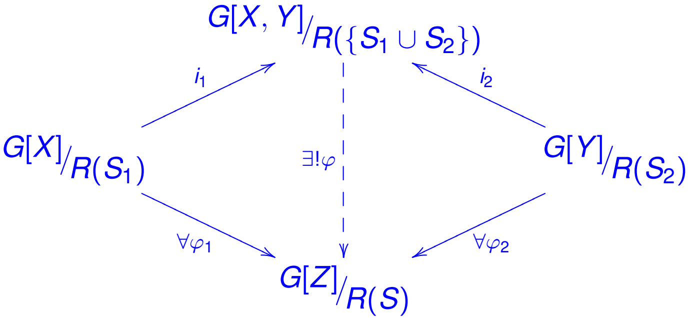


# Examples

Consider the category of coordinate $G$-groups
$Coord_G = (coordinate\ groups, G - homomorphisms)$.

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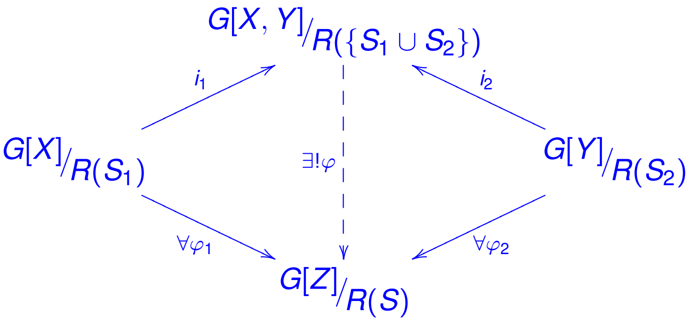


If we consider the subcategory of coordinate groups of irreducible varieties, does there exist the coproduct? How is it defined?

# Key observation

## Proposition

The coproduct exists in the category of coordinate groups of irreducible varieties, i.e. if ${}^{G[X]}/_{R(S_1)}$ and ${}^{G[Y]}/_{R(S_2)}$ are coordinate groups of irreducible varieties then so is the coordinate group ${}^{G[X,Y]}/_{R(\{S_1 \cup S_2\})}$.

## Hint of proof

Using the duality between coodinate groups and varieties: the coproduct corresponds to the direct product of varieties.

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## Hint of proof

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# Completion

- If the group $G$ is countable and equationally Noetherian, then the set of coordinate groups of irreducible varieties is countable:
- $\{G = C_0, C_1, \dots\}$
- We can consider $C_0 < C_0 \sqcup C_1 < \dots$ and define $\mathfrak{U}(G) = \bigsqcup_i C_i$.
- The group $\mathfrak{U}(G)$ has the following properties: it is countable, $G$-discriminated by $G$ and every f.g. $G$-group discriminated by $G$ embeds into it.
- It looks good!
- one minute later...

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- Its structure is not “nice”.
- Sensitive to changes: up to isomorphism, up to isomorphism on a fixed number of generators, up to presentation...
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# Digression while recovering from disapointment

- The coproduct induces a structure of semi-lattice on the set of coordinate groups of irred. varieties.
- When do we have in fact a structure of lattice? In which categories is the coproduct decomposition unique?
- If $C = C_1 \sqcup C_2 \sqcup \cdots \sqcup C_k$ and $C = C'_1 \sqcup C'_2 \sqcup \cdots \sqcup C'_l$, then $k = l$ and upto re-enumeration $C_i \simeq C'_i$
- This is true for abelian groups, fields...
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# Using the language of model theory

- A model $M$ is homogeneous if for any tuples $a = (a_1, \dots, a_k)$ and $b = (b_1, \dots, b_k)$ of elements of $M$ such that $(M, a) \equiv (M, b)$ and for any $c \in M$ there exists $d \in M$ so that $(M, a, c) \equiv (M, b, d)$.
- The reason why the completion is not nice is because it is not homogeneous.
- First, let's "adjust" the definition:
- A model $M$ is $\forall$-homogeneous if for any tuples $a = (a_1, \dots, a_k)$ and $b = (b_1, \dots, b_k)$ of elements of $M$ such that $(M, a) \equiv_\forall (M, b)$ and for any $c \in M$ there exists $d \in M$ so that $(M, a, c) \equiv_\forall (M, b, d)$.
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# The completion (second attempt)

- Set $G_0 = G$ and define inductively $G_i = \mathfrak{U}(G_{i-1})$.
- Define the completion $U(G)$ of $G$ to be the direct limit of $G_i$.

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# Results

### Theorem

*Let $G$ be an equationally Noetherian group. Then the completion $U(G)$ is countable, $G$-discriminated by $G$, and any f.g. group discriminated by $G$ (existentially) embeds into $U(G)$. Re-defining the well-known terms in model theory, the completion $U(G)$ is a $\forall$-homogenous and $\forall$-universal model (and hence $\forall$-saturated).*

### Corollary (c.f. Keisler-Shelah theorem)

$H_1 \equiv_{\forall} H_2$ (in the language of groups enriched with constants from $G$) if and only if $U(H_1) \simeq U(H_2)$.

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# What are completions of particular groups?

- If $G$ is a finite group, then $U(G) = G$.
- If $G$ is a t.f. f.g. abelian group, then $U(G) = \mathbb{Z}^\omega$.
- If $G$ is a free group, then $U(G) = F_\omega^{\mathbb{Z}^\omega}$?
- What about the Lyndon's completion?
- If $G$ is a torsion-free hyperbolic group, then $U(G) = (G * F_\omega)^{\mathbb{Z}^\omega}$?
- What is the completion of your favourite group?

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# Final remarks

- Everything formulated for equationally Noetherian groups can be extended to models of a functional language.
- Instead of f.g. models, analogous completions can be constructed for countable models,etc.
- Similar construction can be done for f.g. (countable,...) models of the q.i. theory.
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THANK YOU!