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Path Following and Boundary Value Problems:
A Continuing Influence in Dynamics
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About the stability of discrete systems with shocks and friction

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Abstract

This talk deals with some properties of the trajectories of systems of particles that are forced to remain on one side of an obstacle, so that the trajectories involve shocks together with contact and friction conditions. Recall that these conditions imply that the differential equations of the dynamics must be understood in the sense of measures and we give some results concerning the set of equilibrium states and the smoothness and the approximation of the trajectories. In particular, the set of equilibrium states for given data may consist of several configurations ranging from a single point to infinitely many solutions.

Next we focus on the stability analysis. Since unilaterality means that the convenient framework for studying the dynamics is not a vector space, and since friction implies dissipativity, the classical stability theorems no longer apply. The first part of the talk consists of analysing the dynamics starting from a neighborhood of an equilibrium state in a classical phase space, and estimating whether the trajectory diverges from the equilibrium or not. This is only carried out for small size systems.

For larger size systems, we observe in particular that, among very large sets of equilibrium solutions, an equilibrium state generically involves some particles strictly stuck by friction, others in imminent sliding, and still others out of contact. For this generic case, a new notion of stability specially adapted to systems with Coulomb friction will be introduced. Whereas classical stability results in mechanics concern perturbations of the initial data in a classical phase space, we study here the effects of a perturbation of the external forces. The main point

of the talk is to back up a conjecture, by closed-form calculations in the case of simple systems and by numerical computations for more complex systems.

The statement of the conjecture is the following:

Let a discrete system with a finite number of degrees of freedom be submitted to nonregularized unilateral contact and Coulomb friction. Assume the data are such that there exists an equilibrium state in which some points are strictly stuck while the others are in imminent sliding. Then, the trajectory produced by any sufficiently small perturbation of the forces leads to a new equilibrium where the number of strictly stuck particles is larger than before the perturbation.

If the system has a nonzero stiffness matrix and the perturbation is constant, then we establish that for small-size systems the final state is reached in finite time and is such that all the particles are strictly stuck.