# Cracked Sets in Image Processing and the Segmentation Problem Revisited

Michel Delfour
delfour@CRM.UMontreal.ca
*Centre de recherches mathématiques et*
*Département de mathématiques et de statistique*
*Université de Montréal*
*C.P. 6128, succ. Centre-ville*
*Montréal, Québec H3C 3J7 (Canada)*

**Abstract**

The study of the problem of linguistic or visual perceptions has been initiated by several pioneering authors such as H. Blum in 1967, D. Marr and E. Hildreth in 1980 (also book of D. Marr in 1982). It involves specialists of psychology, artificial intelligence, and experimentalists such as Hubel and Wiesel in 1962 and Campbell and Robson in 1968. It is a complex multilevel multidisciplinary problem.

The first level of automatic image processing is the detection of the contours or the boundaries of the objects in the image. One of the celebrated formulation of the image segmentation problem was given by Mumford and Shah in 1989. It involves an objective function defined on the whole frame of the image and not only on the contours plus a penalization term on the length of the partitioning curves or interfaces. At coarse scales this formulation tends to neglect long slender objects and does not see cracks.

The object of this talk is to introduce the new family of *cracked sets* which yields a compactness result in the $W^{1,p}$-topology associated with the *oriented distance function* and to give an original application to the celebrated *image segmentation* problem formulated by Mumford and Shah [3]. The originality of the approach is that it does not require a penalization term on the *length of the segmentation* and that, within the set of solutions, there exists one with minimum *density perimeter* as defined by Bucur and Zoésio. This theory can also handle $N$-dimensional images. The talk is completed with several variations of the problem with or without a penalization term on the length of the segmentation. In particular, it revisits and recasts the earlier existence

theorem of Bucur and Zolésio for sets with a uniform bound or a penalization term on the density perimeter in the $W^{1,p}$-framework. If time permits we will indicate some avenues to a numerical approximation of this formulation of the problem via cracked sets.

[1] M.C. Delfour and J.P. Zolésio, *The new family of cracked sets and the image segmentation problem revisited*, Commun. Inf. Syst. **4** (2004), no. 1 , 29- -52.

[2] M.C. Delfour and J.P. Zolésio, *Shape identification via metrics constructed fromthe oriented distance function*, Control Cybernet. **34** (2005), no. 1, to appear.

[3] D. Mumford and J. Shah, *Optimal approximations by piecewise smooth functions and associated variational problems*, Comm. Pure Appl. Math. **42** (1989), no. 5, 577-685.