Mathematical and computational strategies for hybrid stochastic/deterministic systems

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Abstract

Hybrid systems arise as couplings of stochastic models representing active small scales, to deterministic macroscopic equations, and are commonplace in a wide array of applications ranging from catalysis and polymeric flows to stochastic models for tropical and open ocean convection. A major challenge in all these problems arises in the direct numerical simulation of realistic size systems involving both scale and model disparities; furthermore, due to nonlinear interactions across a wide range of scales, the stochasticity inherited from the microscopic model can play a subtle but important role in the dynamic behavior of the overall system.

In this talk we attempt to address directly or indirectly these issues in the context of prototype mathematical hybrid models that capture essential features of their complex counterparts. In particular they allow us to derive computationally inexpensive mesoscopic deterministic models for the average behavior of the hybrid systems in various asymptotic limits, and to develop and test stochastic coarse-graining strategies. A key feature of the simplified prototype models studied here is that they permit computationally feasible detailed comparisons of the derived deterministic and stochastic coarse-grained models against direct numerical simulations of the full hybrid system.

Joint work with A. J. Majda (Courant) and A. Sopasakis (UMass).