Wave capture and wave-vortex duality for internal gravity waves

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Abstract

New and unexpected results are presented regarding the nonlinear interactions between a small-scale wavepacket and a large-scale vortical mean flow, with an eye towards internal wave dynamics in the atmosphere and oceans and the problem of 'missing forces' in atmospheric gravity-wave parametrizations. The present results centre around a wave-breaking scenario termed 'wave capture', which differs significantly from the standard wave-breaking scenarios associated with critical layers or mean density decay. We focus on the peculiar wave-mean interactions that accompany wave capture. Examples of these interactions are presented for layerwise two-dimensional, layerwise non-divergent flows in a three-dimensional Boussinesq system, in the strong-stratification limit.

The nature of the interactions can be summarized in the phrase 'wave-vortex duality', whose key points are firstly that wavepackets behave in some respects like vortex pairs, as originally shown in the pioneering work of Bretherton (1969), and secondly that a collection of interacting wavepackets and vortices satisfies a conservation theorem for the sum of wave pseudomomentum and vortex impulse, provided that the impulse is defined appropriately. It must be defined as the rotated dipole moment of the Lagrangian-mean potential vorticity (PV). This PV differs crucially from the PV evaluated from the curl of either the Lagrangian-mean or the Eulerian-mean velocity. The results are established here in the strong-stratification limit for rotating

(quasi-geostrophic) as well as for non-rotating systems. The concomitant momentum budgets can be expected to be relatively complicated, and to involve far-field recoil effects in the sense discussed in Buhler & McIntyre (2003). The results underline the three-way distinction between impulse, pseudomomentum, and momentum.

While momentum involves the total velocity and pressure fields, impulse and pseudomomentum involve, in different ways, only the vortical part of the velocity field.