Public-Key Cryptosystems from the Worst-Case Shortest Vector Problem

Chris Peikert
Georgia Tech

Computer Security & Cryptography Workshop
12 April 2010

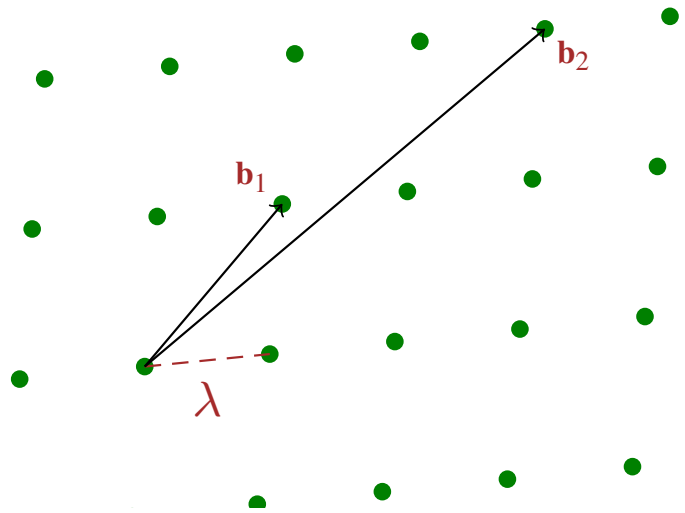
Talk Outline

1. State of Lattice-Based Cryptography
2. Main Result: Public-Key Encryption based on GapSVP
3. Future Work

Shortest Vector Problem(s)

A lattice $\mathcal{L} \subset \mathbb{R}^n$ having basis $\mathbf{B} = \{\mathbf{b}_1, \ldots, \mathbf{b}_n\}$ is:

$$\mathcal{L} = \sum_{i=1}^{n} (\mathbb{Z} \cdot \mathbf{b}_i)$$



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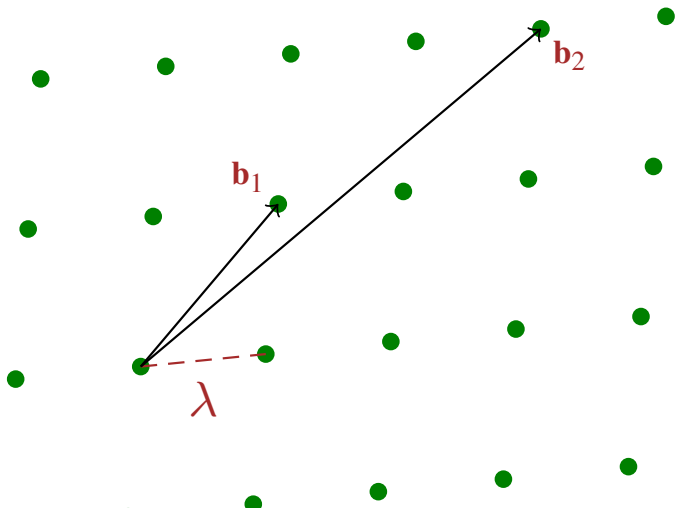

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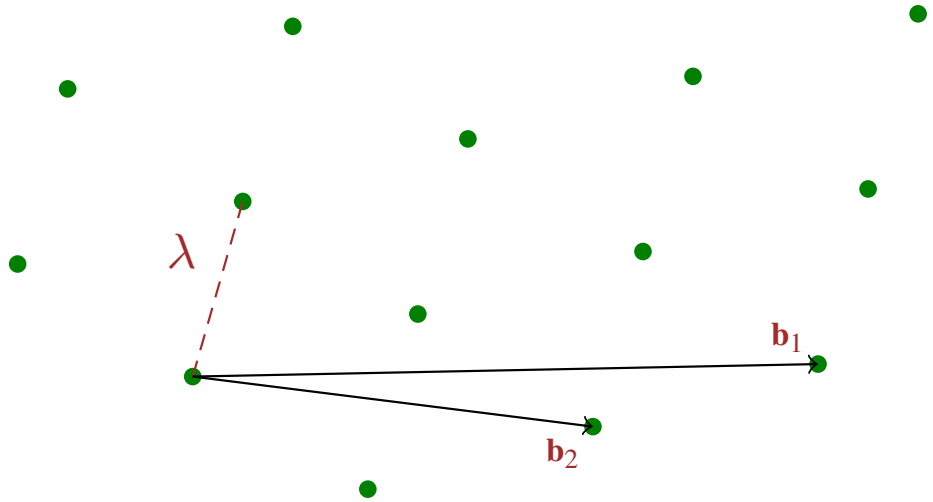

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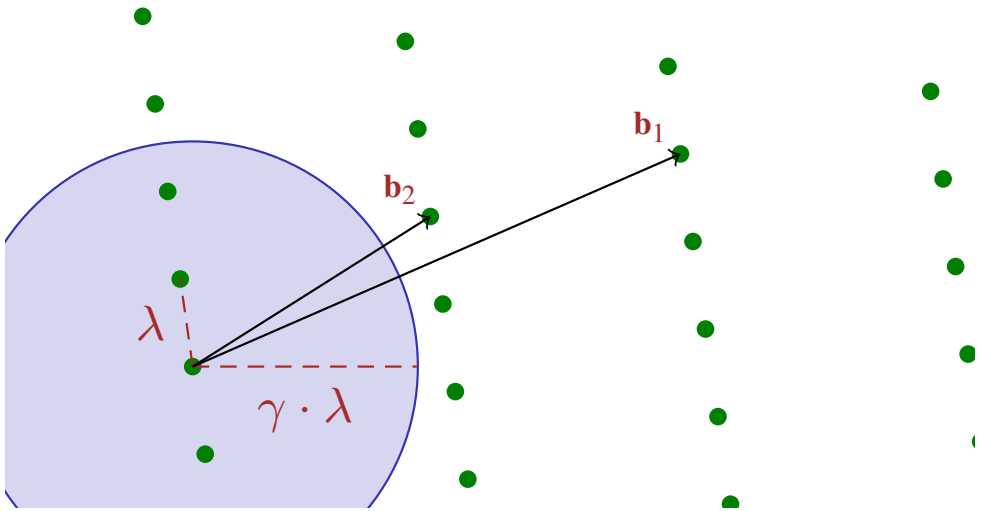

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Unique SVP (γ-uSVP)

- Given $\mathbf{B}$ with 'γ-unique' shortest vector, find it.

Worst-Case Complexity

GapSVP

$\gamma =$	$2^{(\log n)^{1-\epsilon}}$	$\sqrt{n}$	n	$2^{\sim n}$
	NP-hard*	$\in$ coNP	(some) crypto	$\in$ P
	[Ajt98,. . . ,HR07]	[GG98,AR05]	[Ajt96,. . . , MR04,Reg05]	[LLL82,Sch87]

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uSVP

$\gamma =$	??	$\sqrt[4]{n}$	$n^{1.5}$
	NP-hard	$\in$ coAM [Cai98]	crypto [AD97/07,Reg03]

Taxonomy of Lattice-Based Crypto

‘minicrypt’

OWF [Ajt96,. . .]

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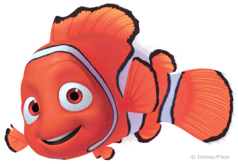

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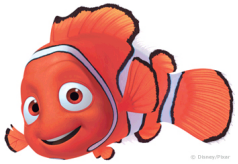

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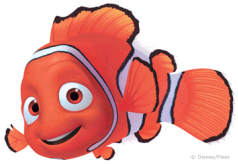

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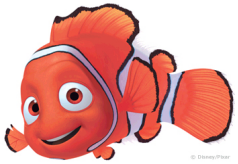

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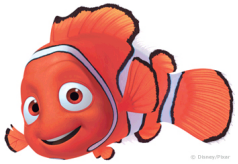

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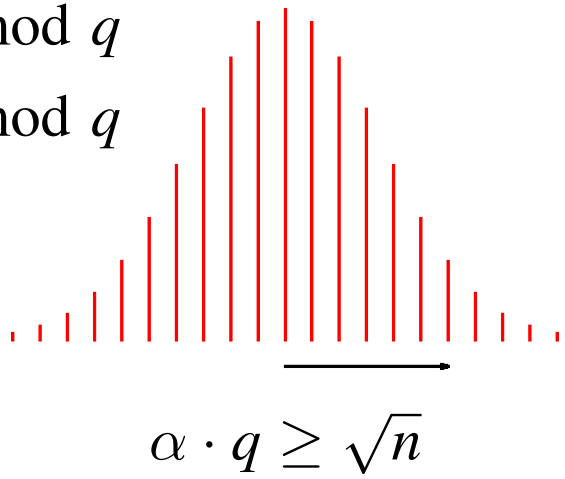

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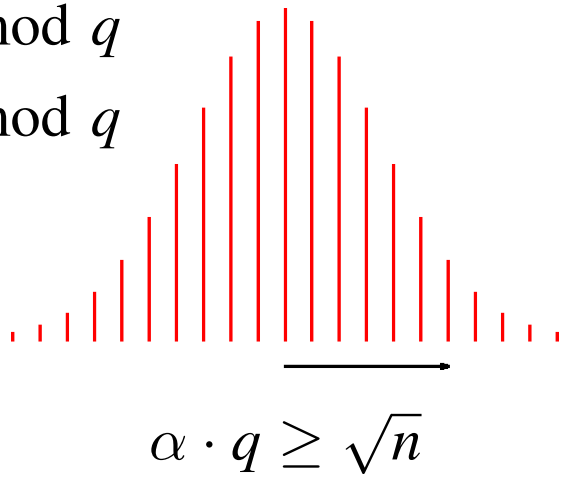

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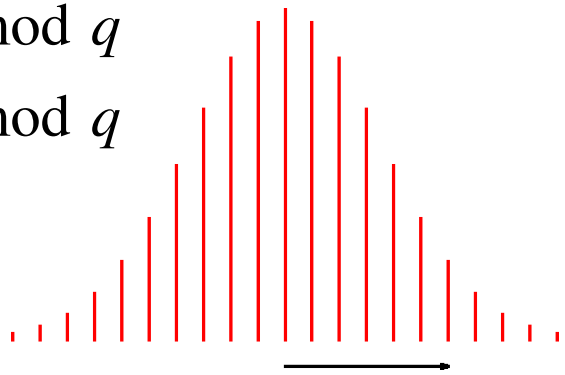

$\alpha \cdot q \geq \sqrt{n}$

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State of the Art

(n/α)-GapSVP etc. $\leq$ search-LWE $\leq$ decision-LWE $\leq$ crypto

quantum [Reg05]

prime $q = \text{poly}(n)$ [BFKL94,R05]

[R05,PW08,GPV08, PVW08,AGV09,ACPS09,...]

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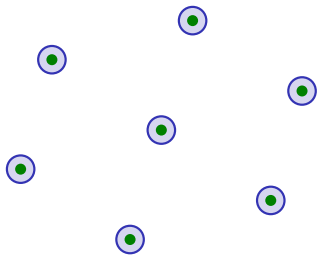
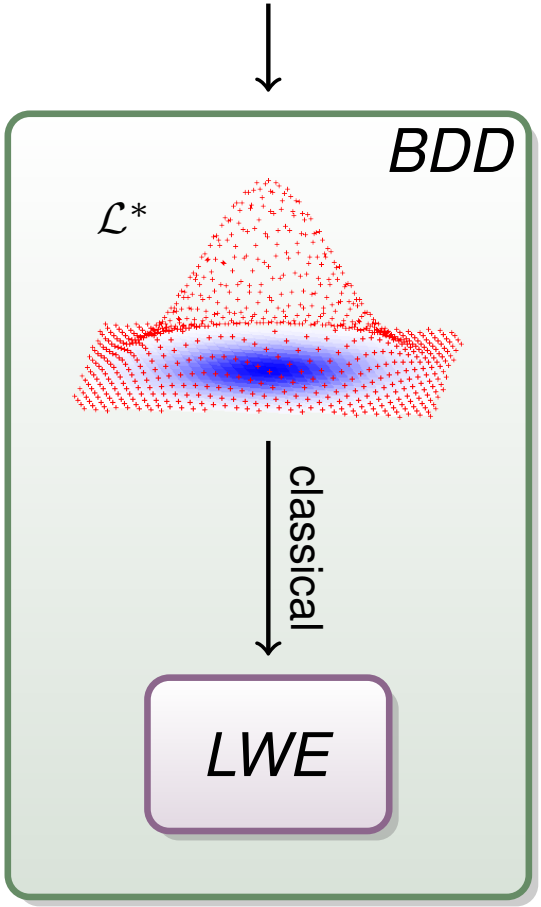
3. **New LWE-based chosen ciphertext-secure encryption**
   - ⋆ Simpler construction, milder assumption than prior CCA [PW08]

[Regev05] Hardness of LWE

BDD on $\mathcal{L}$:
$d \ll \lambda/2$





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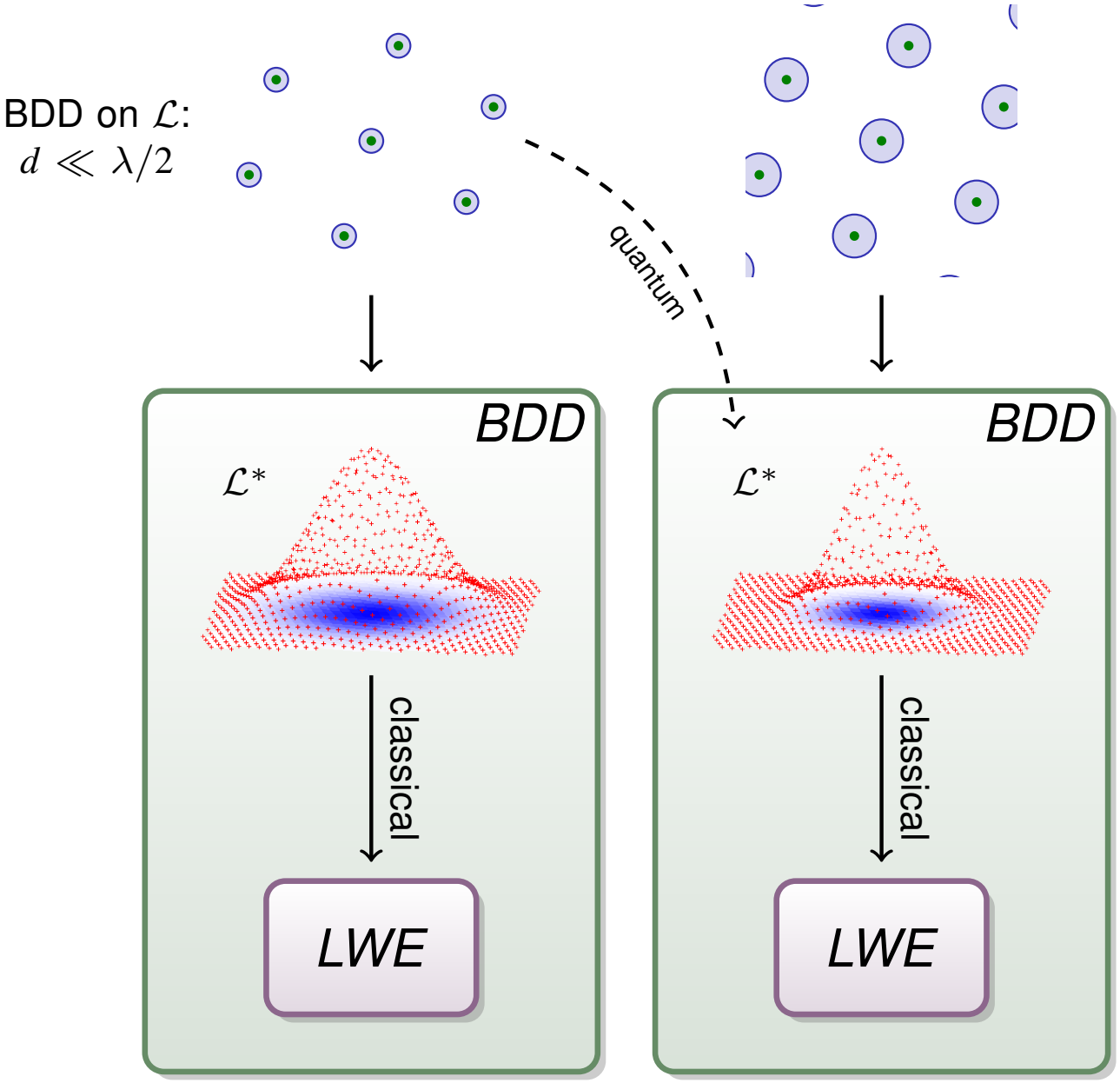

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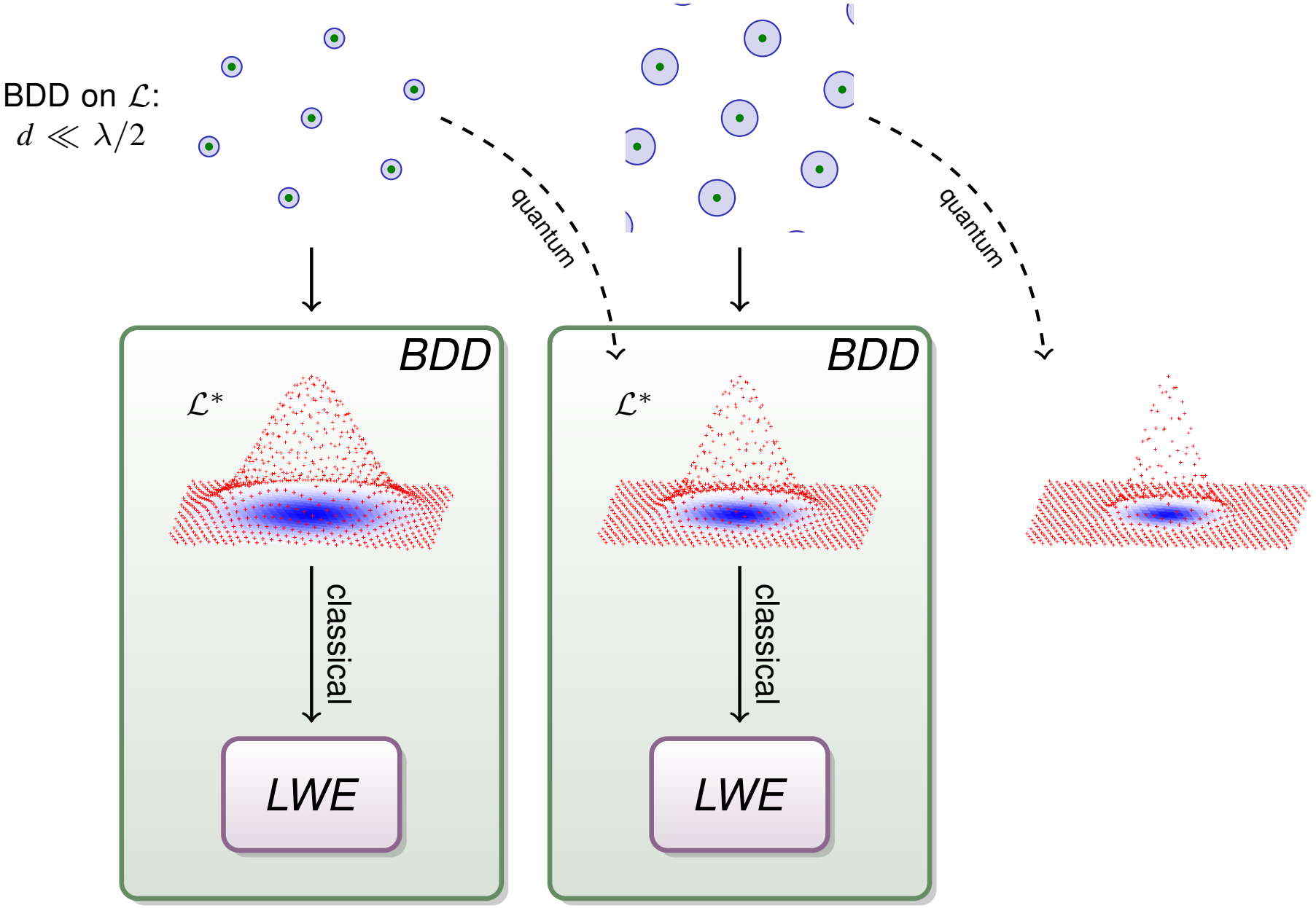

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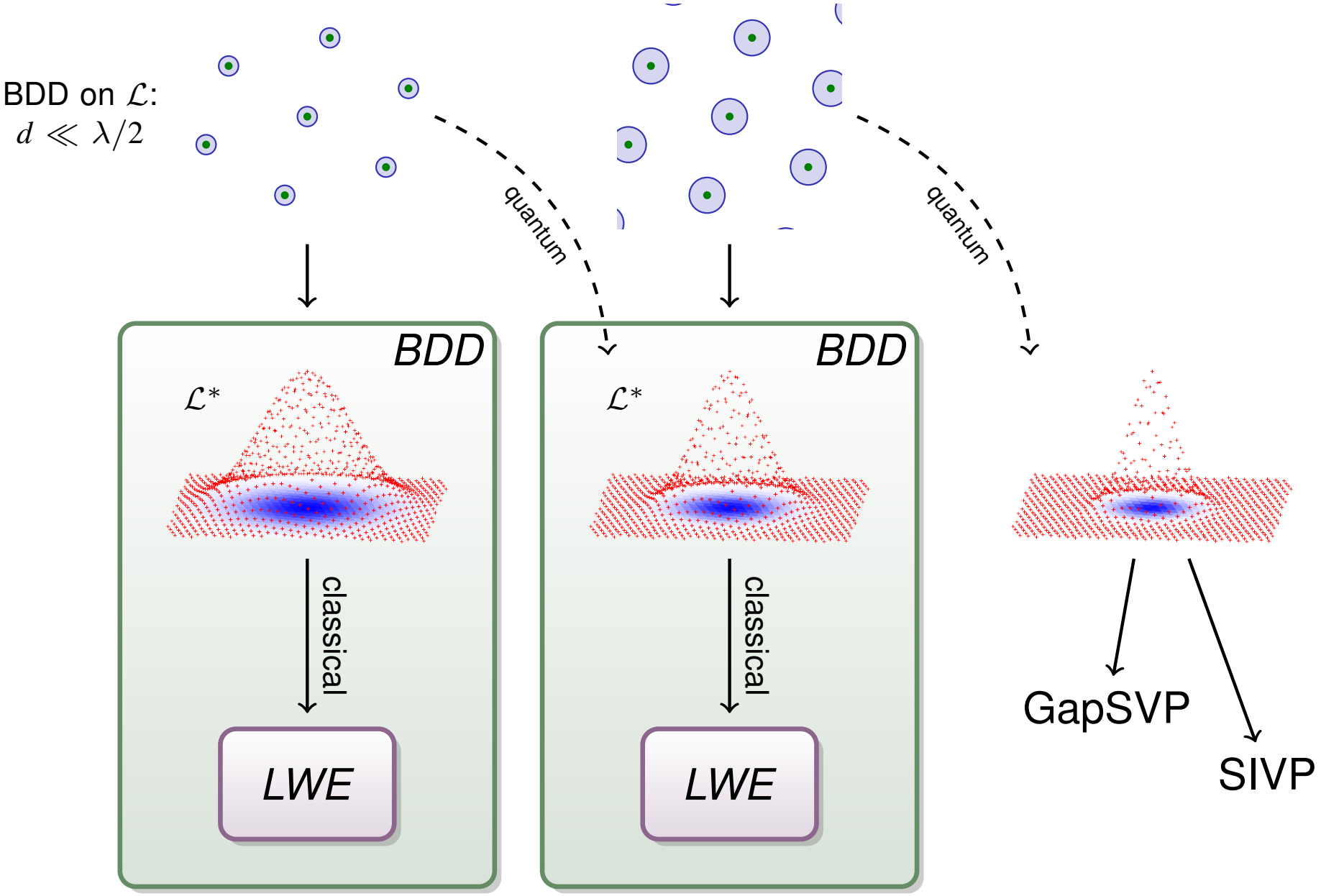
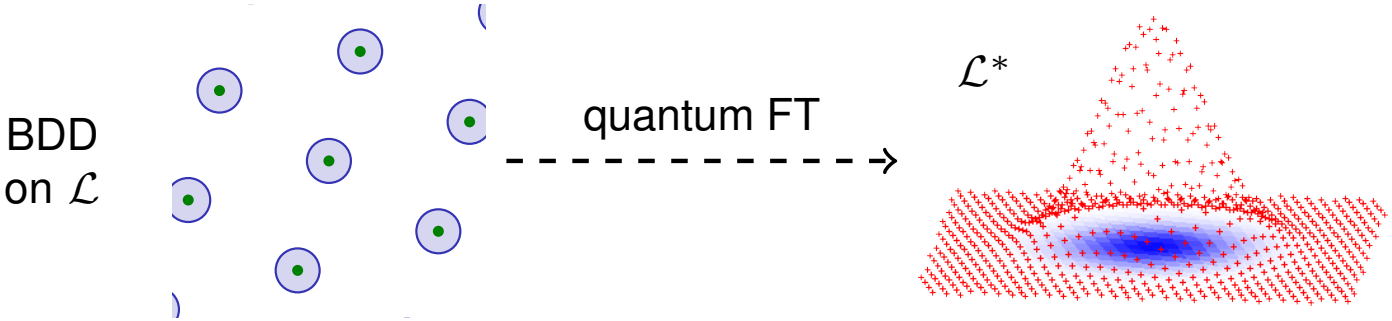


Why Quantum?

- ‘Obvious’ answer: iterative step



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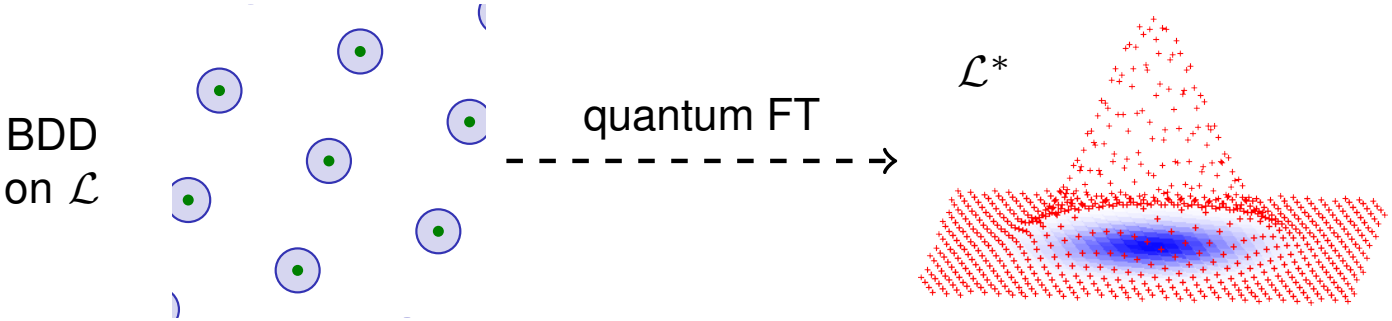

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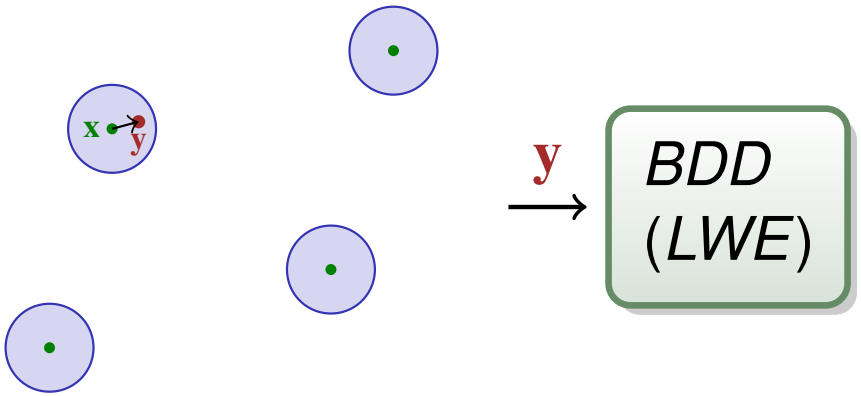

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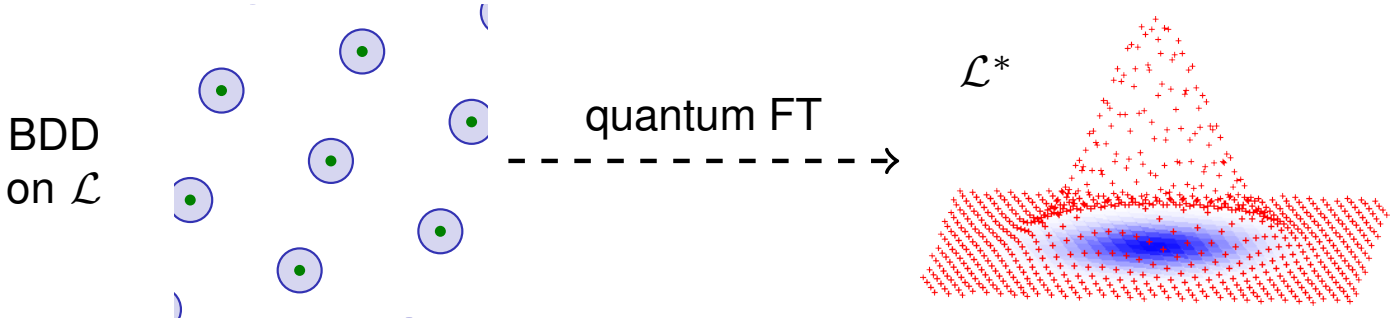

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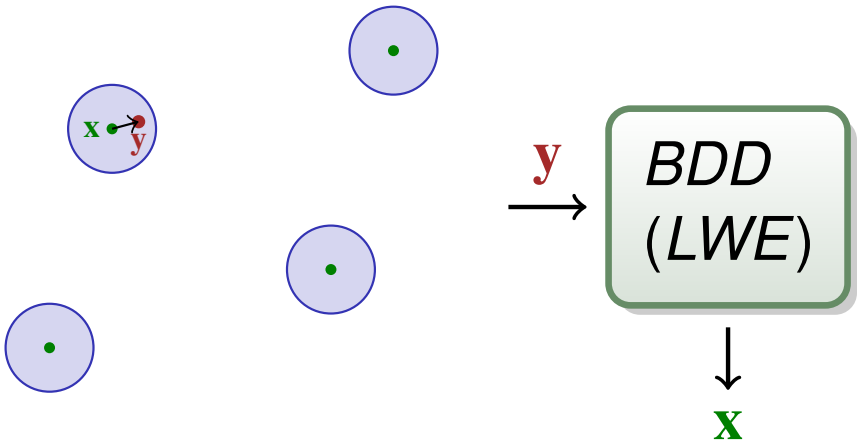

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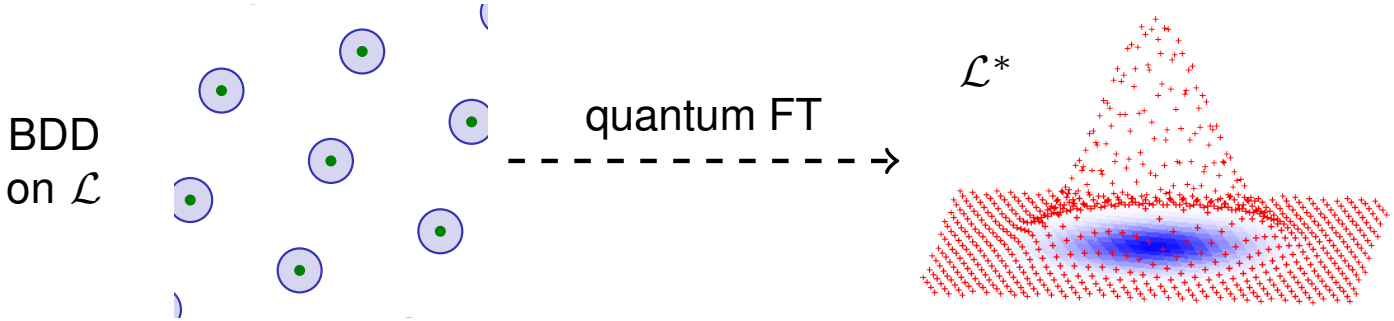

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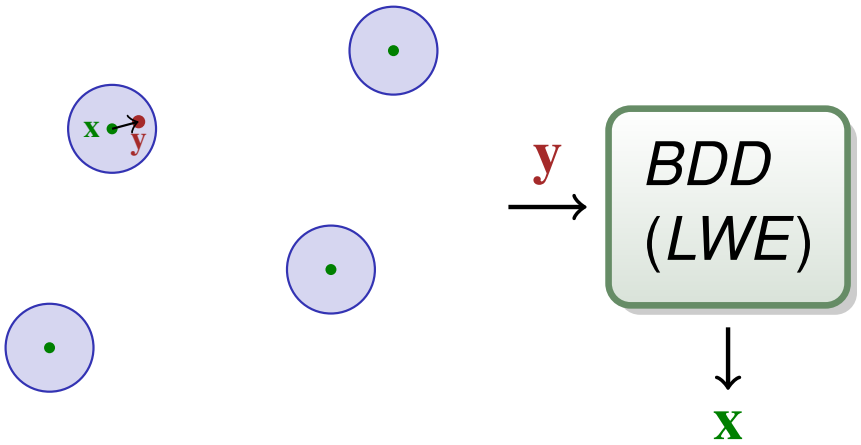
✔ *Quantum can “uncompute”* $\mathbf{x}$



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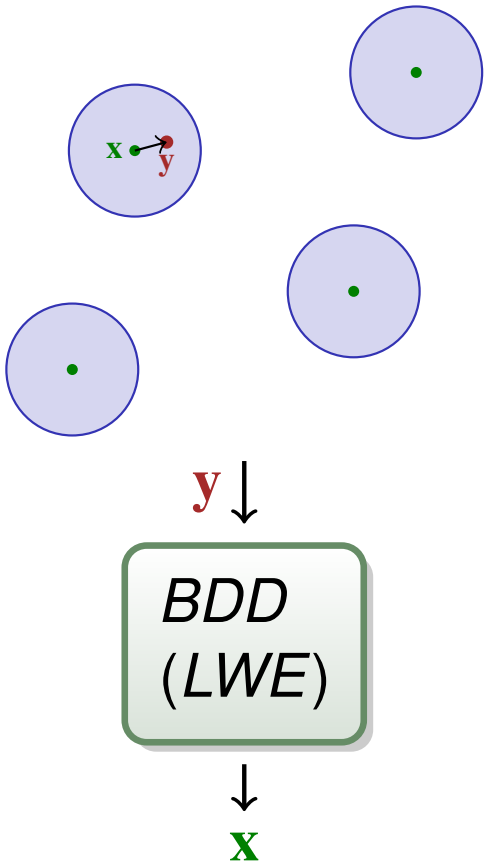

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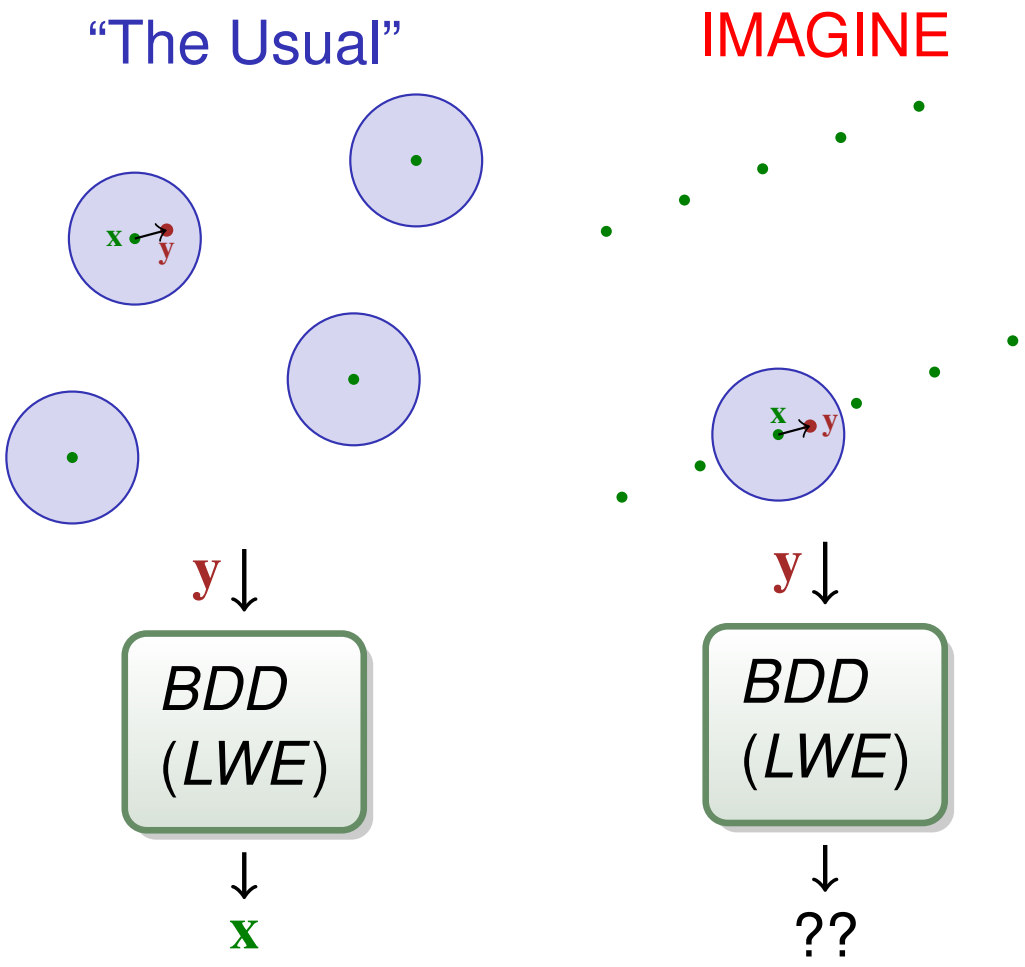

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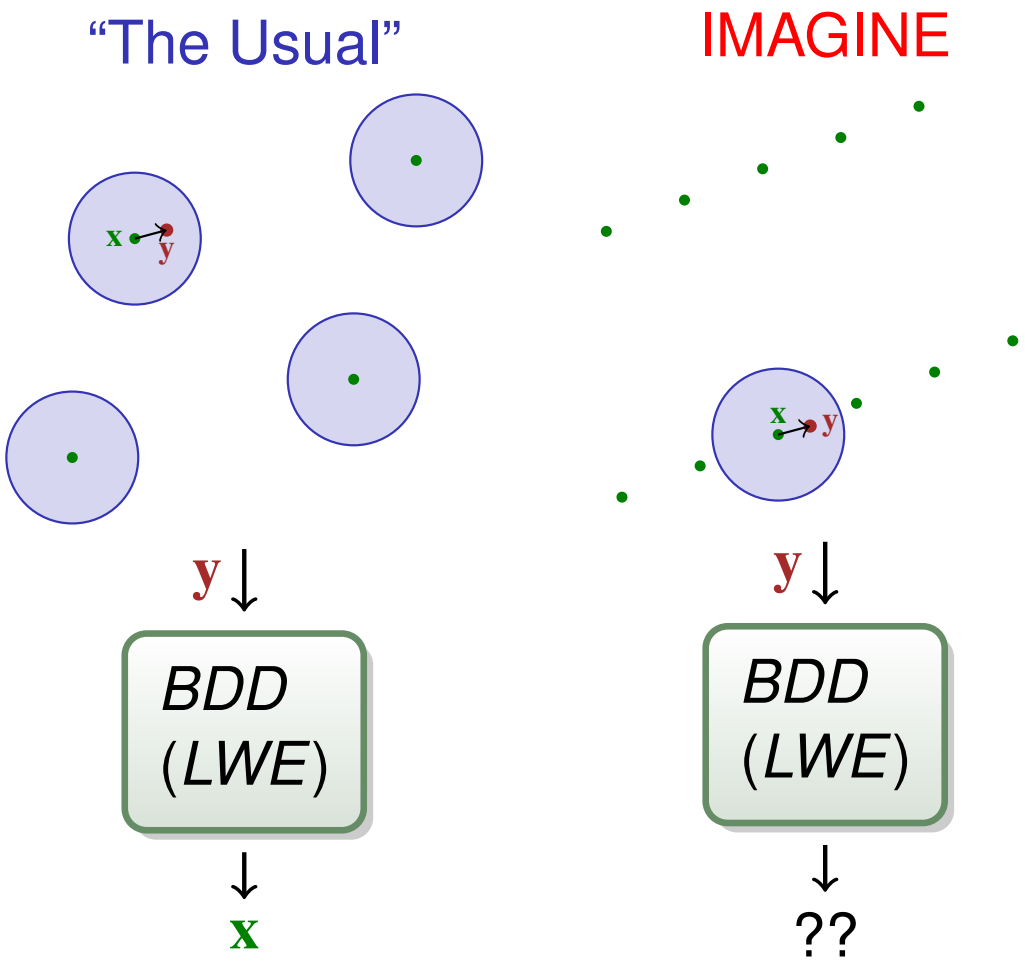

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Incorrect (& unknown!) LWE distribution

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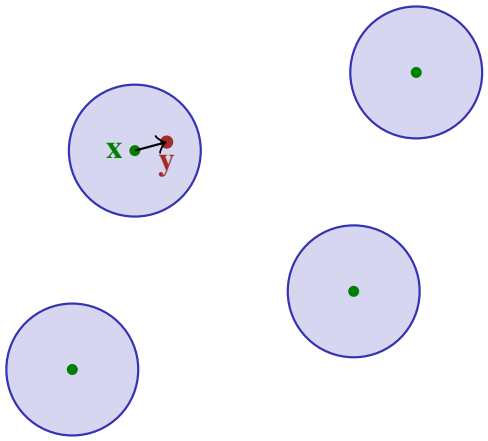

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BDD (LWE)

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$\mathbf{x}$

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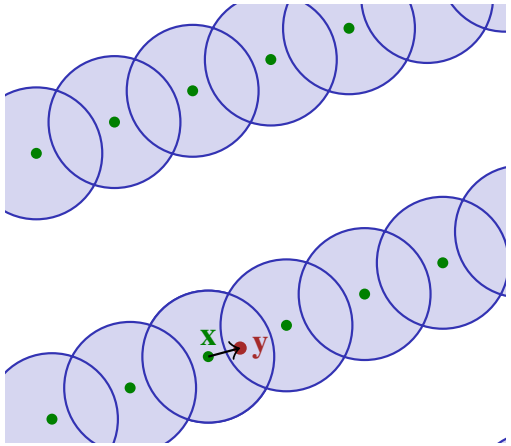

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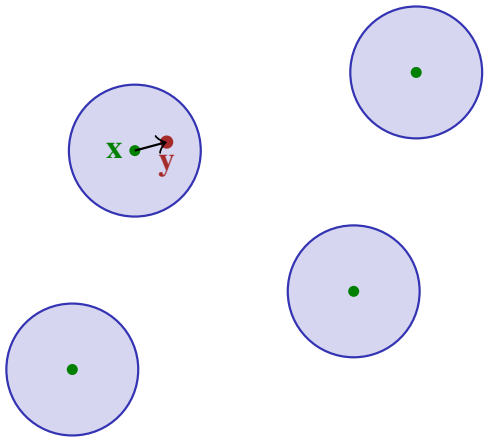

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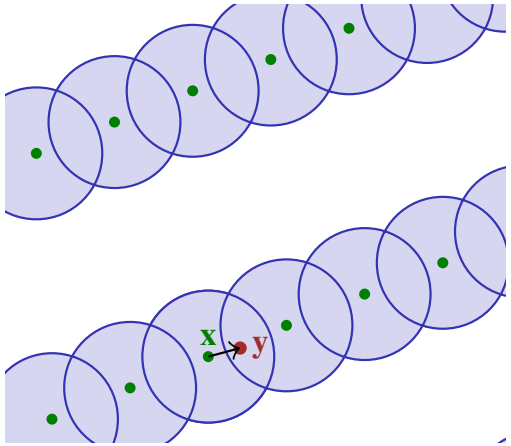

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- View as [GoldGold98] AM proof between reduction and oracle

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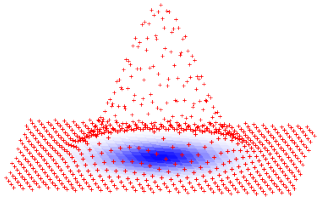
1. What about  in $BDD \rightarrow LWE$ reduction?

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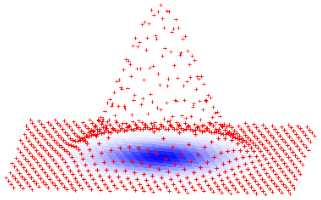
1. What about  in $\boxed{BDD \to \boxed{LWE}}$ reduction?

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- ★ Use [GPV08] sampling algorithm with 'best available' basis for $\mathcal{L}^*$.

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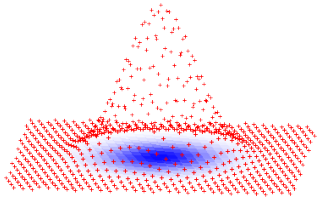
1. What about  in $BDD \to LWE$ reduction?

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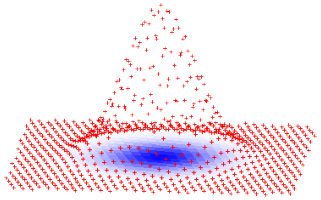
1. What about  in $\boxed{BDD \to \boxed{LWE}}$ reduction?

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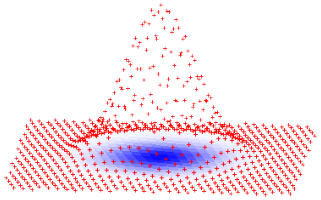
1. What about  in $\boxed{BDD \to \boxed{LWE}}$ reduction?

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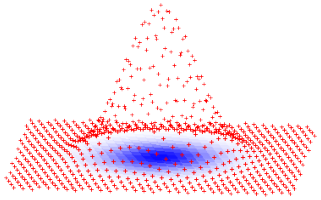
1. What about  in $\boxed{BDD \to \boxed{LWE}}$ reduction?

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   Option 1: crypto directly based on search-LWE

   Option 2: search = decision for 'smooth' q and Gaussian error

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- (NB: for general error dists, hybrid argument over q_i's fails.)

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- Correlation-secure injective TDF $\Rightarrow$ CCA-secure encryption
  But care needed to make $g_{\mathbf{A}}$ "chosen-output secure."

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3. **Open**: complexity of new 'ζ-to-γ'-GapSVP problem?

   NP-hard for nontrivial ζ? Better algorithms?