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Invariant potential theory, derivatives of inner functions, and B^p spaces in the unit ball of $\mathbb{C}^n$

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Let $\mathbb{B}_n$ be the unit ball in $\mathbb{C}^n$ with boundary S. A bounded holomorphic function f on $\mathbb{B}_n$ is called an inner function if

$$\lim_{r\to 1^-} |f(rt)| = 1 \quad \text{for a.e. } t \in S.$$

For $0 < 1$, the space B^p is defined as the space of holomorphic functions f on $\mathbb{B}_n$ for which

$$\|f\|_{B^p} = \int_{\mathbb{B}_n} (1-|z|^2)^{\frac{n}{p}-n-1} |f(z)|\, d\nu(z) < \infty,$$

where ν is normalized Lebesgue measure on $\mathbb{B}_n$. Using techniques and results of potential theory with respect to the Laplace Beltrami operator on $\mathbb{B}_n$, we prove that for $\frac{n}{n+1} < 1$, the radial derivative Rf of an inner function f satisfies

$$Rf \in B^p \iff \int_{\mathbb{B}_n} (1-|z|^2)^{\frac{n}{p}-n-2} |\widetilde{\nabla} f(z)|^q d\nu(z) < \infty \quad \text{for all } q,\ 1 \le q \le 2,$$

where $\widetilde{\nabla}$ denotes the gradient with respect to the Bergman metric on $\mathbb{B}_n$. From this it now follows that for $n \ge 2$, if f is an inner function on $\mathbb{B}_n$ with $Rf \in B^{\frac{2n}{2n+1}}$, then f is constant.

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