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On C^m-extension and reflection of subharmonic functions

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For Runge-type extensions of subharmonic functions we refer the reader to the paper [1] and literature therein, where one can find several different settings of the problem, including ones concerning subharmonic extensions on Riemannian manifolds.

Here we deal with the following question.

For which compact sets X in $\mathbf{R}^N$, $N \geq 3$ (respectively, $N = 2$), and $m \geq 0$, any function $f \in C^m(X)$ subharmonic on the interior of X can be extended to a function F subharmonic and C^m on all of $\mathbf{R}^N$ with the property $\|F\|_{C^m(\mathbf{R}^N)} \leq A\|f\|_{C^m(X)}$ with $A \in (0, +\infty)$ depending only on X, m and N (respectively, $\|F\|_{C^m(\{|\mathbf{x}| \leq r\})} \leq A\|f\|_{C^m(X)}$ for all $r > 0$ with $A \in (0, +\infty)$ depending only on X, m, r) ?

In [2] and [4] it is shown that for $m = 1$ the just formulated property takes place for an arbitrary Lyapunov domain D with connected complement. A wide class of Jordan C^1-smooth domains is constructed, for which the mentioned extension property fails.

In [3] and [5] it is proved that for each $m \in (1, 3)$ the desired property holds for any Lyapunov-Dini (briefly LD-) domain D with connected complement. The corresponding assertion for $m \in [0, 1) \cup [3, +\infty)$ is false even for balls. A localization theorem for C^m-subharmonic extension ($m \in [1, 3)$) from LD-domains (respectively, from any Jordan domain in $\mathbf{R}^2$) was also obtained, as well as some other corollaries, including corresponding assertions on Lip^m-extension of subharmonic functions.

It is planned to discuss more explicitly these results as well as theorems on C^m-harmonic and subharmonic reflection from LD-domains onto their complementary domains.

References

1. *Gauthier P.M.*, Subharmonic extension and approximation. // Canadian Math. Bull. **37** :1 (1994), 46-53.

2. *Melnikov M.S. and Paramonov P.V.*, C^1-extension of subharmonic functions from closed Jordan domains in $\mathbf{R}^2$.// Izvestiya : Mathematics. **68** :6 (2004), 1165–1178.

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3. *Paramonov P.V.*, C^m-extension of subharmonic functions.// Izvestiya : Mathematics. **69** :6 (2005), 1211–1223.

4. *Paramonov P.V.*, C^1-extension and C^1-reflection of subharmonic functions from Lyapunov-Dini domains into $\mathbf{R}^N$.// Sbornik : Mathematics. **199** :12 (2008), 1809–1846.

5. *Paramonov P.V.*, On C^m-extension of subharmonic functions from Lyapunov-Dini domains into $\mathbf{R}^N$. // Math. Notes. **89** :1 (2011), 160–164.

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