≪ Analyse complexe et la théorie du potentiel ≫Un colloque en l'honneur de Paul M. Gauthier et Kohur Gowrisankaran

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Jensen measures in potential theory

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Given an open set Y in $\mathbb{R}^d$ and $x \in Y$, a (positive Radon) measure μ on Y is called a *Jensen measure* for x (with respect to Y), if μ has compact support in Y and $\mu(s) \leq s(x)$, for all superharmonic functions s on Y. The set $J_x(Y)$ of all such measures has been studied in its own right by various authors. In particular, it has been shown that the set $\operatorname{ext} J_x(Y)$ of its extreme measures consists of the Dirac mass ε_x at x and all finely harmonic measures $\mu_x^V = \varepsilon_x^{V^c}$ (obtained by sweeping of ε_x on V^c), where V is a finely open neighborhood of x such that $\overline{V}$ is compact in Y.

In this talk – based on joint work with Ivan Netuka – it will first be shown that $J_x(Y)$ is a simple union of closed faces of the compact convex set $M_x(\mathcal{P}(Y))$. The set $M_x(\mathcal{P}(Y))$ consists of all representing measures μ for x with respect to the convex cone $\mathcal{P}(Y)$ of all continuous real potentials on Y ($\mu(p) \leq p(x)$, for every $p \in \mathcal{P}(Y)$). Since forty years (Mokobodzki 1971) its set $\operatorname{ext} M_x(\mathcal{P}(Y))$ of extreme points is known to be the set of all swept measures ε_x^A, A (finely closed Borel) set in Y.

Even in the general setting of a $\mathcal{P}$-harmonic space Y, this immediately leads to the description of $\operatorname{ext} J_x(Y)$ as above provided Y satisfies a rather weak approximation property for superharmonic functions (which holds for any Y in classical potential theory and – more generally – in the elliptic case, but also for certain sets Y with respect to the heat equation). Equally sufficient is the following transience of Y (which also holds in the classical case) : There exists a harmonic function $h_0 > 0$ on Y such that, for every compact K in Y, there is a superharmonic function $s \geq 0$ on Y which is equal to h_0 on K, but strictly smaller than h_0 at infinity (h_0-*transience*).

Moreover, we shall see that in the classical case, for bounded open sets Y in $\mathbb{R}^d$, $d \geq 2$, $J_x(Y)$ is the set of all compactly supported probability measures $\mu \in M_x(\mathcal{P}(Y))$ if and only if Y is 1-transient. A topological characterization is given by the following *weak regularity property* : Every non-empty closed-open set in the boundary of Y contains regular boundary points.

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