

## Approximation by polyanalytic polynomials and related topics

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In the talk it is planned to discuss the state of the art, some recent results and open questions in the problem of uniform approximability of functions by polyanalytic polynomials on compact subsets of the complex plane. Recall that a function  $f$  is called polyanalytic of order  $n$ ,  $n \in \mathbb{N}$ , or, shorter,  $n$ -analytic in an open set  $U \subset \mathbb{C}$  if  $\partial^n f / \partial \bar{z}^n = 0$  in  $U$ .

In the most general form the problem we are interested in can be formulated as follows : given a compact set  $X \subset \mathbb{C}$ , which conditions on  $X$  are necessary and sufficient in order that each function which is continuous on  $X$  and  $n$ -analytic on its interior can be uniformly on  $X$  approximated by  $n$ -analytic polynomials? In recent decades several interesting results were obtained in connection with this problem. For instance, it was solved for Carathéodory compact sets (see [1,2]). The corresponding approximability criterion is formulated in terms of a special analytic characteristic of the compact set under consideration (the concept of Nevanlinna domain) and does not depend on the order of polyanalyticity. Several sufficient approximability conditions for non Carathéodory compact sets were also obtained in [2,3,4,5,6]. In many instances these conditions have a reductive nature, which means that the approximation on a compact set  $X$  holds whenever we have the approximation on some appropriately chosen compact subsets of  $X$ . Moreover these conditions are often depended not only on topological and analytical properties of  $X$ , but also on order of polyanalyticity.

### References

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