On generalized sparse grids

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Efficient approximations for multivariate functions can be derived from a one-dimensional multilevel representation of a function (hierarchical basis, generating system, (pre)wavelet system, Fourier series, eigenfunction series or simuilar) by a product construction to obtain a multilevel system for the many-dimensional case and a subsequent proper truncation of the resulting series expansion. This leads to so-called sparse grids which promise to cope with the curse of dimension of conventional product discretizations, at least to some extent. First, we discuss regular sparse grids and their approximation and cost complexities.

Besides regular sparse grids, we can construct specific problem-dependent sparse grids by means of an optimization process which exhibit optimal asymptotic complexity estimates for various situations, error norms and regularity assumptions. We present such methods in detail and discuss some applications from optimization and UQ.