How to truncate a PDE problem set in an unbounded domain: the example of the Schrödinger equation

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The goal of this introductory course is to give an overview of the techniques that can be used to truncate the spatial domain on which is set a Partial Differential Equation trying to minimize the effect of the truncation operation. Essentially, we are interested in the absorbing boundary conditions methods and others to treat general problems. The equation that is chosen here is the Schrödinger equation. This equation, which has many applications in physics and engineering, leads to difficult questions concerning the mathematical construction of absorbing boundary conditions as well as their numerical approximation. In particular, even in the simplest case, it is necessary to develop nontrivial solutions to get numerical schemes with suitable properties (like stability).

In this course, we will discuss the following questions with increasing difficulty

- the free space one-dimensional Schrödinger equation,
- how to take into account potentials in 1d,
- how to extend the methods to 1d nonlinear problems,
- two-dimensional problems,
- inclusion of potentials and nonlinearities.

In all these developments, we will show hot to treat the problem from the mathematical analysis to the numerical simulation.

Référence.

X. Antoine, A. Arnold, C. Besse, M. Ehrhardt, A. Schädle, *A Review of Transparent and Artificial Boundary Conditions Techniques for Linear and Nonlinear Schrödinger Equations*, Communications in Computational Physics **4** (4) (2008), pp. 729-796 (free online).