Domain Decomposition of Stochastic PDEs: A Novel Preconditioner and Its Parallel Performance

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A general parallel iterative algorithm is described for efficient solution of the interface (Schur complement) problem arising in the domain decomposition of stochastic partial differential equations (SPDEs). The iterative solver avoids the explicit construction of both the local and global Schur complement matrices. In contrast to simple one dimensional SPDE the interface problem grows rapidly for two dimensional case, and dictates the parallel performance of the algorithm. In this investigation, a stochastic analog of Neumann-Neumann domain decomposition preconditioner is introduced. The implementation of the preconditioner requires a local solve of a stochastic Dirichlet problem followed by the solution of a stochastic Neumann problem in each iteration of the conjugate gradient iterative solver. A parallel graph partitioning tool is used for optimal decomposition of the finite element mesh for load balancing and minimum interprocessor communication. For efficient memory usage and minimum floating point operation, the numerical implementation of the algorithm exploits the multilevel sparsity structure of the coefficient matrix of the stochastic system, namely, the sparsity structure due to the finite element discretization and the block sparsity structure due to the orthogonal representation and projection of the stochastic processes. For numerical demonstration, a two dimensional elliptic SPDE with non-Gaussian random coefficients is tackled. The strong and weak scalability of the algorithm is investigated using Linux cluster.

keywords *Domain decomposition method; Neumann-Neumann preconditioner; Stochastic partial differential equation*

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