Visualization of High-Order Finite Element Methods

Robert M. Kirby

School of Computing and Scientific Computing and Imaging Institute, University of Utah, 50 S. Central Campus Dr., Salt Lake City, UT 84112 USA e-mail: kirby@cs.utah.edu

Abstract

The visualization of high-order methods is an area that is heavily used but often scientifically marginalized. The aim of this talk is to present new research on visualization techniques that both respect and exploit the mathematical structure of high-order methods.

Over the last forty years, tremendous effort has been exerted in the pursuit of numerical methods which are both *flexible* and *accurate*, hence providing sufficient fidelity to be employed in the numerical solution of a large number of models and sufficient quantification of accuracy to allow researchers to focus their attention on model refinement and uncertainty quantification. High-order finite element methods (also known as spectral/*hp* element methods) using either the continuous Galerkin or discontinuous Galerkin formulation have reached a level of sophistication such that they are now commonly applied to a diverse set of real-life engineering problems in computational solid mechanics, fluid mechanics, acoustics and electromagnetics. Unfortunately, there has been little emphasis by the scientific community and, in particular, the visualization community, on providing visualization algorithms and the corresponding software solutions tailored to high-order methods; in particular, almost no research has been done to develop visualization methods based on the *a priori* knowledge that the data was produced by a high-order finite element simulation.

Visualization of computed results is often used as a means of understanding and evaluating the numerical approximation of the mathematical model, and it provides a means of "closing the loop" – that is, of critically evaluating the computational results for refinement of the model and/or numerics or for interpretation of the physical world. Visualizations of high-order finite element results which do not respect the *a priori* knowledge of how the data were produced and which do not provide a quantification of the visual error produced undermine the scientific process just described.

In this talk we describe several efforts at creating accurate and efficient visualization methods for high-order methods. We will present the mathematical foundations and algorithms for ray-tracing high-order methods and for particle-based methods for isosurface extraction, and will discuss a spin-off application in which we generate isosurface triangulations of sampled, volumetric, three-dimensional scalar fields.