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SPACE-TIME DISCONTINUOUS GALERKIN METHODS FOR COMPRESSIBLE FLOWS

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Abstract

The accurate solution of the compressible Navier-Stokes equations on a domain with time-dependent boundaries requires moving and deforming meshes. In general, it is non-trivial to obtain a conservative and accurate numerical discretization on this type of meshes which motivated the development of a space-time discontinuous Galerkin finite element method. In this numerical technique no distinction is being made between space and time variables and the equations are discretized directly in four dimensional space. This approach then provides optimal flexibility to deal with time-dependent boundaries and deforming elements and naturally results in a conservative scheme even on deforming locally refined meshes with hanging nodes. The space-time DG algorithm combines the well-know benefits of the compact stencil of DG methods, such as optimal flexibility for mesh adaptation and parallel computing, with a fully conservative arbitrary Eulerian Lagrangian approach suitable for dynamic meshes.

In this short course an overview of the space-time DG method for the compressible Navier-Stokes equations will be given. Also, applications to other fluid flow problems will be discussed. The course will start with a discussion of the basic aspects of space-time discretizations and their analysis using the advection-diffusion equation as a model problem. Next, the extension to the Navier-Stokes equations will be discussed. Special emphasis will be put on the efficient solution of the large number of coupled nonlinear algebraic equations resulting from the space-time discretization. These equations are solved using a pseudo-time integration method in combination with a multigrid technique. The pseudo-time integration is performed with specially designed Runge-Kutta time integration methods which act as smoother in the multigrid algorithm. Also, results of

a two-level smoothing analysis will be discussed. This combined pseudo-time integration-multigrid method provides an efficient solution technique which maintains the locality of the DG scheme and does not result in the large computational overhead of frequently used Newton methods. Another topic which will be discussed is the choice of variables, e.g. conservative, primitive or entropy variables. These variables have for instance different properties when considering the limit to incompressible flows.

We will illustrate the space-time DG algorithm with a number of fluid dynamics applications, for instance the flow around a 3D delta wing on an adapted mesh and the dynamic stall of an airfoil in rapid pitch up maneuver.