

# A randomized algorithm for rank revealing QR factorizations and applications

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## Abstract

The basic steps of a RRQR Factorization are: (i) select columns from the input matrix  $A$ , (ii) permute them to leading positions to a new matrix  $A_p$ , (iii) compute a QR Factorization  $A_p = QR$ , (iv) reveal  $\text{rank}(A)$  from  $R$ . Since their introduction [1, 2], algorithmic trends have involved procedures for deterministically selecting columns from  $A$  [3, 4, 5]. Motivated by recent results in theoretical computer science [6, 7, 8] we present a novel algorithm for randomized column selection. Following work in [9] we illustrate our algorithm for approximation of stock market related matrices.

## References

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