Atelier ≪ Information quantique : codes, géometrie et structures aléatoires ≫
24–26 octobre 2011

**Workshop on Quantum Information: Codes, Geometry and Random Structures
October 24–26, 2011**

Polar codes for classical, private, and quantum communication

Mark Wilde *
mwilde@gmail.com

Channel polarization is a phenomenon in which a particular recursive encoding induces a set of synthesized channels from many instances of a memoryless channel, such that a fraction of the synthesized channels become near perfect for data transmission and the other fraction become near useless for this task. The channel polarization effect then leads to a simple scheme for data transmission : send the information bits through the perfect channels and "frozen" bits through the useless ones. One contribution of the present work is to leverage several known results from the quantum information literature to demonstrate that the channel polarization effect takes hold for channels with classical inputs and quantum outputs. We construct linear polar codes based on this effect, and we also demonstrate that a quantum successive cancellation decoder works well, by exploiting Sen's recent "non-commutative union bound" that holds for a sequence of projectors applied to a quantum state. In addition, consider that Mahdavifar and Vardy have recently exploited the channel polarization phenomenon to construct codes that achieve the symmetric private capacity for private data transmission over a degraded wiretap channel. We build on their work and demonstrate how to construct quantum wiretap polar codes that achieve the symmetric private capacity of a degraded quantum wiretap channel with a classical eavesdropper. Due to the Schumacher–Westmoreland correspondence between quantum privacy and quantum coherence, we can construct quantum polar codes by operating these quantum wiretap polar codes in superposition. Our scheme achieves the symmetric coherent information rate for quantum channels that are degradable with a classical environment. This condition on the environment may seem restrictive, but we show that many quantum channels satisfy this criterion, including amplitude damping channels, photon-detected jump channels, dephasing channels, erasure channels, and cloning channels. Our quantum polar coding scheme has the desirable properties of being channel-adapted and symmetric capacity-achieving.

* School of Computer Science, McGill University, 3480 rue University, Montréal, QC H3A 2A7, CANADA.