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WORKSHOP ON QUANTUM INFORMATION: CODES, GEOMETRY AND RANDOM STRUCTURES
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Universal low-rank matrix recovery from Pauli measurements

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We show that there exists a universal set of $O(rd \log 6d)$ Pauli measurements, from which one can reconstruct any unknown matrix M of rank r and dimension d. In particular, M can be recovered by solving a convex program (minimizing the trace or nuclear norm). This has applications to compressed sensing methods for quantum state tomography. It improves on previous results in two respects : it uses a fixed set of Pauli measurements, independent of the unknown quantum state ; and it implies nearly-optimal bounds on the error of the reconstructed density matrix. In addition, this method can be used to learn arbitrary (not necessarily low-rank) states, up to an error of $O(1/r)$ in 2-norm. Similar results hold for measurements in any incoherent operator basis.

Our main technical contribution is to show that the random Pauli sampling operator obeys the "restricted isometry property" (RIP) over the set of all low-rank matrices. Our proof uses Dudley's entropy bound for Gaussian processes, together with known bounds on covering numbers of the trace-norm ball.

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