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# Nodal lines for random eigenfunctions of the Laplacian on the torus

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## Abstract

We study the volume of nodal sets for eigenfunctions of the Laplacian on the standard torus in two or more dimensions. We consider a sequence of eigenvalues  $4\pi^2\lambda$  with growing multiplicity  $N \rightarrow \infty$ , and compute the expectation and variance of the volume of the nodal set with respect to a Gaussian probability measure on the eigenspaces. We show that the expected volume of the nodal set is  $\text{const}\sqrt{\lambda}$ . Our main result is that the variance of the volume normalized by  $\sqrt{\lambda}$  is bounded by  $O(1/\sqrt{N})$ , so that the normalized volume has vanishing fluctuations as we increase the dimension of the eigenspace.

**This work is joint with Zeev Rudnick.**