

Workshop “Mathematical aspects of quantum chaos”
Atelier «Aspects mathématiques du chaos quantique»
June 2-6, 2008/2 au 6 juin 2008

Counting nodal lines which touch the boundary of an analytic domain

John A. Toth
Department of Mathematics and Statistics
McGill University
Burnside Hall, 805 Sherbrooke W.
Montréal (Québec), H3A 2K6
CANADA

jtoth@math.mcgill.ca

Abstract

We consider the zeros on the boundary $\partial\Omega$ of a Neumann eigenfunction ϕ_λ of a real analytic plane domain Ω . We prove that the number of its boundary zeros is $O(\lambda)$ where $-\Delta\phi_\lambda = \lambda^2\phi_\lambda$. We also prove that the number of boundary critical points of either a Neumann or Dirichlet eigenfunction is $O(\lambda)$. It follows that the number of nodal lines of ϕ_λ (components of the nodal set) which touch the boundary is of order λ . This upper bound is of the same order of magnitude as the length of the total nodal line, but is the square root of the Courant bound on the number of nodal components in the interior. More generally, the results are proved for piecewise analytic domains.

Joint work with Steve Zelditch.