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Entropy of quantum limits for the 2-torus

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Abstract

Quantized linear automorphisms of the torus, or “cat maps", are interesting toy models of quantum chaos. In particular, the classification of semiclassical measures (or “quantum limits") arising from this model is a difficult problem, though some partial results are known.

For example, on the 2-torus, a result of Faure–Nonnenmacher shows that any quantum limit with an atomic component of weight α must also have a Lebesgue component of weight $\geq \alpha$. Recent methods of Anantharaman and Nonnenmacher can be used to show that the entropy of a quantum limit must be at least half of $h_{\max}$, the entropy of Lebesgue measure (and the maximal entropy for the system).

We combine these into the following result: If a quantum limit for the 2-torus has weight α on ergodic components of small entropy (say, less than $\varepsilon h_{\max}$), then it must also have weight α on components of high entropy (i.e., greater than $(1-\varepsilon)h_{\max}$). Roughly speaking, for every “localized" component of the measure, there is a “delocalized" component of equal mass.