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A lower bound for nodal count on discrete and metric graphs

Gregory Berkolaiko
Department of Mathematics
Texas A&M University
College Station, TX 77843-3368
USA

`Gregory.Berkolaiko@math.tamu.edu`

Abstract

According to a well-know theorem by Sturm, a vibrating string is divided into exactly N nodal intervals by zeros of its N-th eigenfunction. Courant showed that one half of Sturm's theorem for the strings can be carried over to the theory of membranes: N-th eigenfunction cannot have more than N domains. He also gave an example of an eigenfunction high in the spectrum with a minimal number of nodal domains, thus excluding the existence of a non-trivial lower bound.

An analogue of Sturm's result for discretizations of the interval was discussed by Gantmacher and Krein. The discretization of an interval is a graph of a simple form, a chain-graph. But what can be said about more complicated graphs? It has been known since the early 90s that the nodal count for the Schrödinger operator on quantum trees (where each edge is identified with an interval of the real line and some matching conditions are enforced on the vertices) is exact too: zeros of the N-th eigenfunction divide the tree into exactly N subtrees.

We discuss extensions of this result to general graphs (non-trees) for both continuous and discrete Schrödinger operator. We show that the number of nodal domains of the N-th eigenfunction is bounded below by $N - L$, where L is the number of links that distinguish the graph from a tree (defined as the dimension of the cycle space or the rank of the fundamental group of the graph). This existence of a lower bound is a reminder of the differences between the graphs and domains in R^d.